Phys141 – Wed 11/29

TODAY: Chapter 18 standing waves

Next HW due Wed

office hours TODAY only until 12.45pm

Superposition Principle

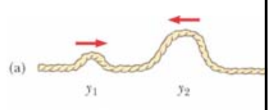
Two pulses travel in opposite directions

- Wave function of the pulse moving to the right: $y_1(x,t)$
- Wave function of the pulse moving to the left: $y_2(x,t)$

Pulses have the same speed along x (set by material parameters)

Total wave:

$y(x,t)=y_1(x,t)+y_2(x,t)$



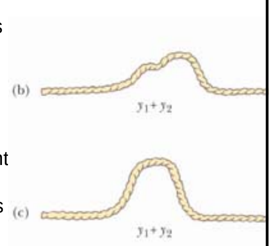
Superposition – interference of waves

Waves start to overlap, the resultant wave function remains

$y(x,t)=y_1(x,t)+y_2(x,t)$

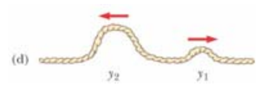
When the crests meet the resultant wave has a larger amplitude than either of the original waves

(constructive INTERFERENCE)



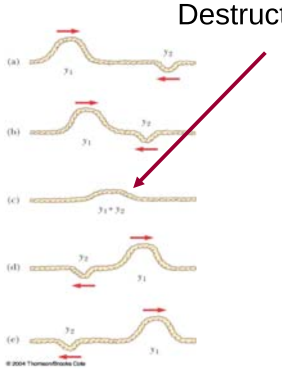
Superposition

- pulses separate
- Continue moving in their original directions
- Pulse shapes remain unchanged



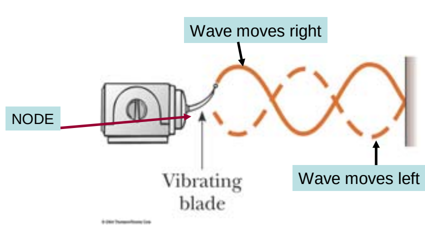
DEMO shive machine

Destructive Interference



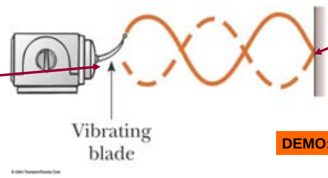
Example

Interference between incoming and reflected signal -> Standing waves



Standing Wave on a String

NODE



Reflection+
Inversion
FIXED end

DEMO: standing waves

Standing waves between two driven or fixed ends

Example: Standing waves created by interference of sine waves

Add right moving and left moving sine wave

$y_1(x,t) = A\sin\left(kx - \omega t\right)$ $y_2(x,t) = A\sin\left(kx + \omega t\right)$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$
$$y(x,t) = A\sin\left(kx - \omega t\right) + A\sin\left(kx + \omega t\right)$$
$$y(x,t) = 2A\sin(kx)\cos(\omega t)$$

There is no *kx – wt* term, therefore it's not a traveling wave.
Maximum amplitude depends on x position.
Amplitude oscillates in time.

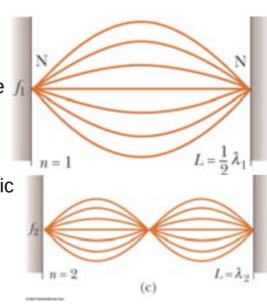
Standing Waves: normal modes

First normal mode (on right):
There are nodes at both ends
There is one antinode in the middle
This is the longest wavelength mode this cavity of length L

$\lambda = 2L$

The normal modes are called **harmonics:** λ=L second harmonic (add one node each harmonic)

Frequencies Of harmonics: $f_n = n\dfrac{v}{2L} = \dfrac{n}{2L}\sqrt{\dfrac{T}{\mu}}$



Quantization of allowed frequencies

Standing Waves

Analyzing $y = (2A \sin kx) \cos \omega t$

- Every element in the medium oscillates in simple harmonic motion with the same frequency, ω
- The amplitude of the simple harmonic motion is $(2A \sin kx)$ - it depends on the location x
  – A point x of zero amplitude is called a **node**

$$x_n = \frac{n\lambda}{2} \qquad n = 0, 1, \ldots$$

  – A point x of maximum displacement amplitude, 2*A* is called an **antinode**

$$x_m = \frac{\lambda}{4} + \frac{m\lambda}{2} \qquad m = 0, 1, \ldots$$

Review of forced Oscillations

Driving force: $F_o \sin(\omega t)$

For damped harmonic motion:

$-k\,x - bv_x + F_o \sin(\omega t) = ma_x$

Amplitude of a driven oscillation (ω_0 natural frequency of undamped oscillator):

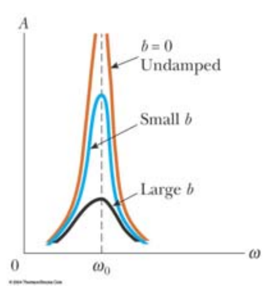
$$A = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

Amplitude of oscillation - resonance

Timing is everything!

One needs to give the pendulum the next push when it is back at its original position, not too early and not too late

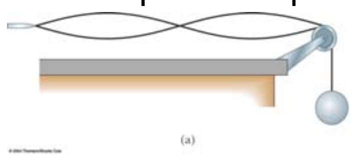
SHATTERED GLASS DEMO



Resonance

- When the frequency of the driving force is near the natural frequency ($\omega \approx \omega_0$) a dramatic increase in amplitude occurs: so called ***resonance***
- Natural frequency ω_0 is the resonance frequency
- **At resonance: Applied force is 90º out of phase with position, but in phase with the velocity**
  - The power delivered is **F** · **V**
  - Power is a maximum in this case since **F** and **v** are in phase

Standing Wave on a String, Example Set-Up

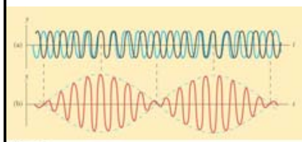


- One end of the string is attached to a vibrating blade
- The other end passes over a pulley with a hanging mass attached to the end
  - This produces the tension in the string
- The string is vibrating in its second harmonic

Resonance

- A system is capable of oscillating in one or more normal modes
- If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system

Beat Frequency



$$A_{resultant} = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t$$

equals the difference between the frequencies of two waves