

Phys141 – Mon 11/27

TODAY/Wed: Ch 18 – superposition of waves skip 18.5 and 18.8

Next HW due WED

Pre-class quiz due Fri

Office hours this week Wed only until 1pm or set up time via email

What energy is stored in a sinusoidal wave?

We can model each element of a string as a simple harmonic oscillator

- Oscillation in the y -direction

Every element of length Δx has the same total energy (μ mass per unit length; $\mu \Delta x$ mass in an element)

$$\Delta K = \frac{1}{2} (\mu \Delta x) v_y^2$$

$$\Delta K = \frac{1}{2} (\mu \Delta x) \omega^2 A^2 \cos^2(kx - \omega t)$$

Kinetic Energy in one wavelength (Integral of $\cos^2 = \frac{1}{2}$)

$$K_\lambda = \frac{1}{4} \mu \lambda \omega^2 A^2$$

Same amount of energy stored in potential energy

$$E_\lambda = \frac{1}{2} \mu \lambda \omega^2 A^2$$

Is energy transferred in a sinusoidal wave?

- The power is the rate at which the energy is being transferred:

$$P = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2} \mu \lambda \omega^2 A^2}{T} = \frac{1}{2} \mu \omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
 - Frequency squared
 - Square of the amplitude
 - Wave speed

Example: 16.40

Power transmitted by a Sound Wave

Power transmitted by sound wave to an area of size A

$$\phi = \frac{\Delta E}{\Delta t} = \frac{E_{\lambda}}{T} = \frac{1}{2} \rho A v \omega^2 s_{\max}^2$$

s_{\max} : Amplitude of longitudinal displacement of air

v: Speed of sound wave

A: area in which total sound wave power is measured

NEW FOR SOUND WAVES
-> compute Power per unit area (Intensity) instead

Intensity of a Sound Wave

Intensity of a wave: **power per unit area**

– Power transmitted through a unit area, A, perpendicular to the direction of the wave

$$I = \frac{\phi}{A}$$

Doppler Effect, Observer Moving

- The frequency heard by the observer, f' , appears higher when the observer approaches the source

$$f' = \left(\frac{v + v_o}{v} \right) f$$

- The frequency heard by the observer, f' , appears lower when the observer moves away from the source

$$f' = \left(\frac{v - v_o}{v} \right) f$$

Doppler Effect, General

Motions of the observer and the source

$$f' = \left(\frac{v + v_o}{v - v_s} \right) f$$

Demo doppler ball

The signs of v_o and v_s depend on the direction of the velocity

- A positive value is used for motion of the observer or the source *toward* the other
- A negative value is used for motion of one away from the other

The Doppler effect is common to all waves, where source or observer can move (radar, laser, sonar)

The magnitude of frequency change does not depend on distance between observer and source, only on their relative velocities

Superposition Principle

Two pulses travel in opposite directions

- Wave function of the pulse moving to the right: $y_1(x,t)$
- Wave function of the pulse moving to the left: $y_2(x,t)$



Pulses have the same speed (set by material parameters) but different shapes

Total wave:

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

Superposition – interference of waves

Waves start to overlap, the resultant wave function remains

$$y(x,t) = y_1(x,t) + y_2(x,t)$$



When the crests meet the resultant wave has a larger amplitude than either of the original waves (constructive INTERFERENCE)



Superposition Example, final

- pulses separate
- Continue moving in their original directions
- Pulse shapes remain unchanged

(d)

DEMO shive machine

Destructive Interference

(a) (b) (c) (d) (e)

$y_1 + y_2$

Example

Standing Wave on a String

DEMO: standing waves

Standing waves are set up by a continuous superposition of waves incident on and reflected from the ends

Example from Lab: Standing waves created by interference of sine waves

Add right moving and left moving sine wave

$$y_1(x,t) = A \sin(kx - \omega t) \quad y_2(x,t) = A \sin(kx + \omega t)$$

$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$$y(x,t) = A \sin(kx - \omega t) + A \sin(kx + \omega t)$$

$$y(x,t) = 2A \sin(kx) \cos(\omega t)$$

There is no $kx - \omega t$ term, therefore it's not a traveling wave.
 Maximum amplitude depends on x position.
 Amplitude oscillates in time.

Standing Waves

Analyzing $y = (2A \sin kx) \cos \omega t$

- Every element in the medium oscillates in simple harmonic motion with the same frequency, ω
- The amplitude of the simple harmonic motion is $(2A \sin kx)$ - it depends on the location x
 - A point x of zero amplitude is called a **node**

$$x_n = \frac{n\lambda}{2} \quad n = 0, 1, \dots$$

- A point x of maximum displacement amplitude, $2A$ is called an **antinode**

$$x_m = \frac{\lambda}{4} + \frac{m\lambda}{2} \quad m = 0, 1, \dots$$

Standing Waves: normal modes

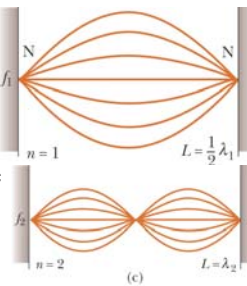
First normal mode (on right):

There are nodes at both ends
 There is one antinode in the middle
 This is the longest wavelength mode
 this cavity of length L

$$\lambda = 2L$$

The normal modes are called **harmonics**: $\lambda = L$ second harmonic (add one node each harmonic)

Frequencies
 Of harmonics: $f_n = n \frac{v}{2L} = \frac{n}{2L} \sqrt{\frac{T}{\mu}}$



Quantization of allowed frequencies
