Phys141 – Fri 10/14 – Lecture 18

- **Today: Chapter 9: center of mass**
  Start of Chapter 10 – rotational motion

- **Administrative:**

Signup sheet available if you forgot your clicker

Lab: Make-up week – you only need to come if you missed one lab. PLEASE CHECK WITH YOUR TA if you are not sure whether you missed a lab!!

The Center of Mass

Center of mass is the point that moves as if all of the mass of the system was concentrated at that point

The system will move as if an external force were applied to a single particle of mass *M* located at the center of mass

– *M* is the total mass of the system

Lets assume for now that a center of mass point exists:

Velocity of the center of mass of a system of particles:

$$\mathbf{v}_{\text{CM}} = \frac{d\mathbf{r}_{\text{CM}}}{dt}$$

Total linear Momentum:

$$M\mathbf{v}_{\text{CM}} = \mathbf{p}_{\text{tot}}$$

Acceleration of the center of mass:

$$\mathbf{a}_{\text{CM}} = \frac{d\mathbf{v}_{\text{CM}}}{dt}$$

Forces In a System of Particles

Forces: change in momentum $\Sigma\mathbf{F}_{ext} = \frac{d\mathbf{p}_{cm}}{dt}$

Forces: Center of mass acceleration

$$\Sigma\mathbf{F}_{ext} = \frac{d}{dt}M\mathbf{v}_{cm} = M\frac{d\mathbf{v}_{cm}}{dt} = M\mathbf{a}_{cm}$$

Center of mass at rest if no net force

Does such a point exist?

System of particles: Center of Mass

The center of mass position $\mathbf{r}_{CM}$ can be defined as:

$$\mathbf{r}_{CM} = \frac{\sum_i m_i \mathbf{r}_i}{M}$$

$\mathbf{r}_i$ is the position of the i th particle (which is of mass m_i)

M is the total mass of all objects

Does COM give the right equation for the velocity, momentum, acceleration, Newton's law?

Velocity of the center of mass of a system of particles: $\mathbf{v}_{CM} = \frac{d\mathbf{r}_{CM}}{dt} = \frac{\sum_i m_i \mathbf{v}_i}{M}$

Total linear Momentum: $M\mathbf{v}_{CM} = \sum_i m_i \mathbf{v}_i = \sum_i \mathbf{p}_i = \mathbf{p}_{tot}$

Acceleration of the center of mass: $\mathbf{a}_{CM} = \frac{d\mathbf{v}_{CM}}{dt} = \frac{\sum_i m_i \mathbf{a}_i}{M}$

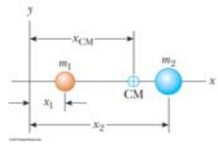
Newton's 2nd law $\Sigma\mathbf{F}_{ext} = \frac{d}{dt}\mathbf{p}_{tot} = \frac{d}{dt}M\mathbf{v}_{cm} = M\frac{d\mathbf{v}_{cm}}{dt} = M\mathbf{a}_{cm}$

Center of Mass, Example

Both masses are on the *x*-axis

The center of mass is on the *x*-axis

The center of mass is closer to the particle with the larger mass

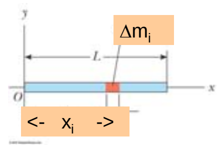


$$x_{cm} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

Center of Mass, Rod

Find the center of mass of a rod of mass *M* and length *L*

The location is on the *x*-axis ($y_{CM} = z_{CM} = 0$)



$$x_{cm} = \frac{1}{M}\sum_i x_i \Delta m_i$$

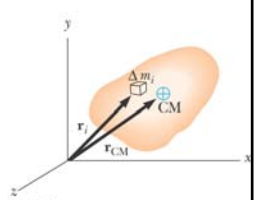
$$x_{cm} = \frac{1}{M}\int x dm = \frac{1}{M}\int_0^L x\frac{M}{L}dx = \int_0^L \frac{x}{L}dx = \frac{1}{2}\frac{L^2}{L} = \frac{L}{2}$$

Example

Demonstration: Walking the plank…

Center of Mass, Continuous object

- An extended object can be considered a distribution of small mass elements, Δm
- The center of mass is located at position $\mathbf{r}_{CM}$



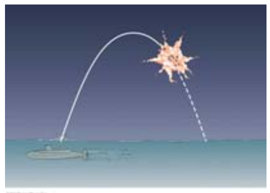
$$\mathbf{r}_{CM} = \frac{1}{M}\int \mathbf{r}\, dm$$

- Or: $x_{CM} = \frac{1}{M}\int x\, dm \quad y_{CM} = \frac{1}{M}\int y\, dm \quad z_{CM} = \frac{1}{M}\int z\, dm$

Internal forces do NOT change center of mass motion!

- A projectile in the air suddenly explodes
- No explosion -> projectile would follow the dotted line

<u>Conservation of Center of Mass momentum:</u> After the explosion, the center of mass of the fragments still follows the dotted line



Demonstration: Airtrack spring coupled gliders

Chapter 10: Rigid Object rotation

Rigid object: - the relative locations of all particles making up the object remain constant

(1) Kinematics
(2) Energy
(3) Forces

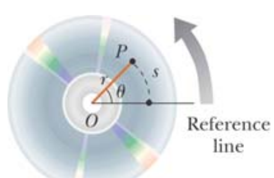
} for rigid, rotating objects

Includes rotation of a point object or spatially extended object

Reminder: Angular Position

- Fixed reference line
- *P* is located at (r, θ)
- arc length s
- radius *r*

$s = \theta r$



Angular Displacement, Angular Speed

Angular displacement $\Delta\Theta$

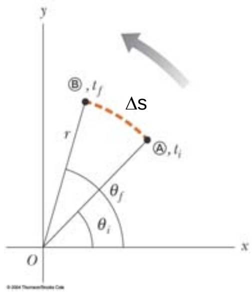
$$\Delta\theta = \theta_f - \theta_i$$

$\Delta s = \Delta\Theta\ r$

angular speed, ω

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Units of ω: radians/sec or s^{-1} (NOTE: radians have no dimension)



Acceleration

angular acceleration $\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

Units of α: rad/s² or s^{-2}

In terms of "real" tangential acceleration: $a_t = r\alpha$

In addition: Centripetal acceleration $a_C = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$

-> Total acceleration (note the directions of angular acceleration and centripetal acceleration are perpendicular)

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

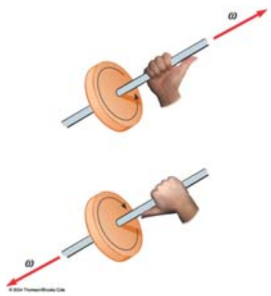
Rotational Kinematic Equations

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

Rotation axis

- The speed and acceleration (ω, α) are the magnitudes of the velocity and acceleration vectors
- The rotation axis is given by the right-hand rule



Kinetic Energy of rotation

Each particle is in motion and has a kinetic energy of

$K_i = ½\, m_i v_i^2$

- Since the tangential velocity depends on the distance, *r*, from the axis of rotation, we can substitute $v_i = \omega_i\, r$
- The total rotational kinetic energy of the rigid object is the sum of the energies of all its particles

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

- Where *I* is called the moment of inertia

Rolling vs sliding demo