Phys141 – Fri 11/18

TODAY: Ch 16, Waves, Ch 17 Sound

Pre-class quiz – next Quiz due Mon

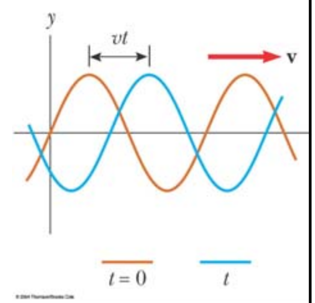
Next HW due MONDAY after thanksgiving

Example: Sinusoidal Waves

For a wave moving at velocity *v* to the right:

$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$

Example 16.13 (start)



Wave Function, Another Form

Wave moves one wavelength λ in one period T:
vT = λ or $v = \lambda / T$

Plug into wave function $y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$

We can define the angular wave number (or just wave number), *k* $k = \frac{2\pi}{\lambda}$

The angular frequency can also be introduced again $\omega = \frac{2\pi}{T}$

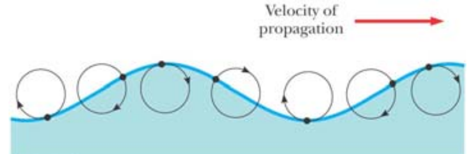
-> $y(x,t) = A\sin(kx - \omega t)$

Longitudinal, transverse and complex waves

Some waves exhibit a combination of transverse and longitudinal waves

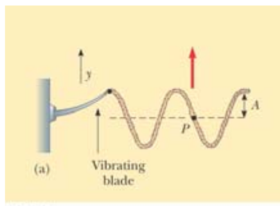
-> points on the wave move back and forth ***and*** up and down simultaneously

-> Thus the motion of the points may look like a circle (see surface water wave example)



How to create sinusoidal waves on a String

- Each element of the string oscillates vertically with simple harmonic motion
  - For example, point *P*



Velocity of point on a Sinusoidal Wave

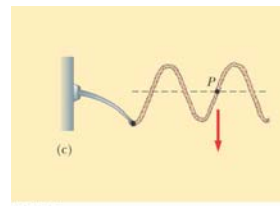
Speed of a fixed point x_o:

$$v_y = \left.\frac{dy}{dt}\right]_{x=x_0}$$

$v_y = -\omega A \cos(kx - \omega t)$

NOT the propagation speed of the wave!

Other example: Jammed wave in traffic jam moves backward. Individual point (car) in traffic jam moves forward

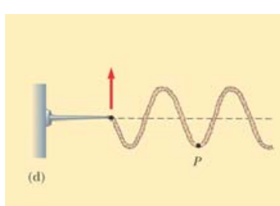


Acceleration in a Sinusoidal Wave

- transverse acceleration of elements:

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}}$$

$a_y = -\omega^2 A \sin(kx - \omega t)$



What energy is stored in a sinusoidal wave?

- We can model each element of a string as a simple harmonic oscillator
  - The oscillation will be in the *y*-direction
- Every element of length Δx has the same total energy (μ mass per unit length)
  $\Delta K = ½ (\mu \Delta x)\, v_y^2$
  $\Delta K = ½ (\mu \Delta x)\, \omega^2 A^2 \cos^2(kx - \omega t)$
- Kinetic Energy in one wavelength
  (Integral of $\cos^2 = ½$)
  $K_\lambda = ¼ \mu \lambda\, \omega^2 A^2$
- Same amount of energy stored in potential energy

Is energy transferred in a sinusoidal wave?

- The power is the rate at which the energy is being transferred:

$$\wp = \frac{\Delta E}{\Delta t} = \frac{\frac{1}{2}\mu\omega^2 A^2 \lambda}{T} = \frac{1}{2}\mu\omega^2 A^2 v$$

- The power transfer by a sinusoidal wave on a string is proportional to the
  - Frequency squared
  - Square of the amplitude
  - Wave speed

Generalizing from sinusoidal waves

Oscillations: $\frac{d^2x}{dt^2} = -\omega^2 x \longrightarrow x(t) = A\cos(\omega t + \phi)$

differential equation --> Solution

For waves: $\frac{\partial^2 y}{\partial t^2} = \left(?\frac{\omega}{k}\right)^2 \frac{\partial^2 y}{\partial x^2} \longrightarrow y(x,t) = A\sin(kx - \omega t)$

linear wave equation

Wave velocity? $v = \frac{\lambda}{T} = \frac{\frac{2\pi}{k}}{\frac{2\pi}{\omega}} = \frac{\omega}{k} \longrightarrow \frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$

Wave Speeds

Wave speed in general $v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$

For a string: $v = \sqrt{\frac{T}{\rho}}$

The speed of compression waves in a material:

Elastic property: compressive force per unit volume
-> bulk modulus B

Inertial forces: proportional to mass of material per unit volume
-> density ρ

$$v = \sqrt{\frac{B}{\rho}}$$

Sound waves

Compression waves of air

Audible waves by the human ear

– Range: 20 Hz to 20,000 Hz

Infrasonic waves <20Hz

Ultrasonic waves > 20,000 Hz

Speed of Sound in Gases

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma_s P}{\rho}}$$

Gases	
Hydrogen (0°C)	1 286
Helium (0°C)	972
Air (20°C)	343
Air (0°C)	331
Oxygen (0°C)	317

Speeds are in m/s

Speed of Sound in liquids

Liquids at 25°C	
Glycerol	1 904
Seawater	1 533
Water	1 493
Mercury	1 450
Kerosene	1 324
Methyl alcohol	1 143
Carbon tetrachloride	926

Speeds are in m/s

Power transmitted by a Sound Wave

Power transmitted by sound wave to an area of size A

$$\wp = \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T} = \frac{1}{2}\rho A v(\omega s_{\max})^2$$

s: longitudinal displacement of air

v: Speed of sound

A: area in which total sound wave power is measured

Intensity of a Sound Wave

Intensity of a wave: **power per unit area**

– Power transmitted through a unit area, *A*, perpendicular to the direction of the wave

$$I = \frac{\wp}{A}$$