Phys141 – Fri 10/14 – Lecture 18

- **Today: Chapter 10: rotational Motion**

- **Administrative:**

Reading for Mon: Chapter 11.1-2

Lab: Make-up week – you only need to come if you missed one lab. PLEASE CHECK WITH YOUR TA if you are not sure whether you missed a lab!!

Chapter 10: Rigid Object rotation

Rigid object: - the relative locations of all particles making up the object remain constant

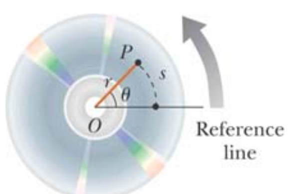
(1) Kinematics
(2) Energy
(3) Forces
} for rigid, rotating objects

Includes rotation of a point object or spatially extended object

Reminder: Angular Position

- Fixed reference line
- *P* is located at (r, θ)
- arc length s
- radius *r*

$s = \theta r$



Angular Displacement, Angular Speed

Angular displacement $\Delta\Theta$

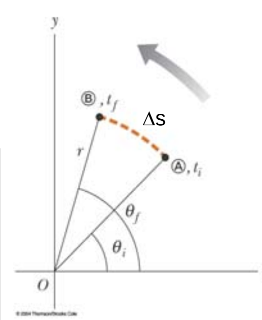
$$\Delta\theta = \theta_f - \theta_i$$

$\Delta s = \Delta\Theta\ r$

angular speed, ω

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

Units of ω: radians/sec or s^{-1} (NOTE: radians have no dimension)



Acceleration

angular acceleration (tangential acceleration)

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

Units of α: rad/s² or s^{-2}

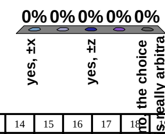
In addition: Centripetal acceleration $a_C = \frac{v^2}{r} = r\omega^2$

-> Total acceleration (note the directions of angular acceleration and centripetal acceleration are perpendicular)

$$a = \sqrt{a_t^2 + a_r^2} = \sqrt{r^2\alpha^2 + r^2\omega^4} = r\sqrt{\alpha^2 + \omega^4}$$

Consider a uniformly rotating object (see blackboard). If the object's angular velocity is a vector (in other words, it points in a certain direction in space) is there a particular direction we should associate with the angular velocity?

1. yes, $\pm x$
2. yes, $\pm y$
3. yes, $\pm z$
4. yes, some other direction
5. no, the choice is really arbitrary



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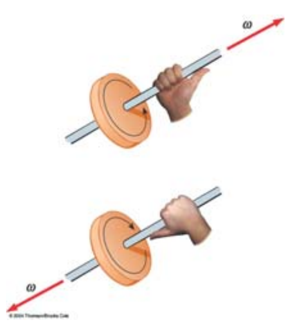
Rotation axis

- The speed and acceleration (ω, α) are the magnitudes of the velocity and acceleration vectors
- The rotation axis is given by the right-hand rule



Rotational Kinematic Equations

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

Kinetic Energy of rotation

Each particle is in motion and has a kinetic energy of

$K_i = ½ m_i v_i^2$

- Since the tangential velocity depends on the distance, *r*, from the axis of rotation, we can substitute $v_i = \omega_i r$
- The total rotational kinetic energy of the rigid object is the sum of the energies of all its particles

$$K_R = \sum_i K_i = \sum_i \frac{1}{2} m_i r_i^2 \omega^2$$

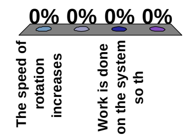
$$K_R = \frac{1}{2}\left(\sum_i m_i r_i^2\right)\omega^2 = \frac{1}{2} I \omega^2$$

- Where *I* is called the moment of inertia

Rolling vs sliding demo

What happens when the volunteer pulls the barbell towards her/him

1. The speed of rotation increases because of conservation of energy
2. The speed of rotation decreases, because of conservation of energy
3. Work is done on the system so the speed of rotation decreases
4. The rotation speed is unchanged



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Moment of Inertia

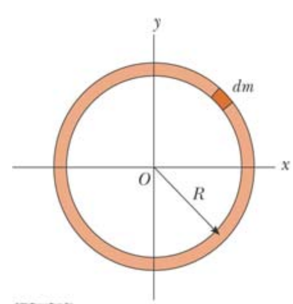
- Definition of moment of inertia: $I = \sum_i r_i^2 m_i$
- Dimensions: ML^2
- SI units: kg·m²
- Calculation: $I = \lim_{\Delta m_i \to 0} \sum_i r_i^2 \Delta m_i = \int r^2 dm$

Moment of Inertia of a Uniform Thin Hoop

- Since this is a thin hoop, all mass elements are the same distance from the center

$$I = \int r^2 dm = R^2 \int dm$$

$$I = MR^2$$



Moment of inertia in terms of densities

Calculate inertia by integrating over length, area, or volume instead of mass:

$$I = \int r^2 dm$$

Volumetric Mass Density
mass per unit volume:

$$\rho = \frac{m}{V}$$

$$I = \int \rho r^2 dV$$

Linear Mass Density
mass per unit length of a rod of uniform cross-sectional area A:

$$\lambda = \frac{m}{L} = \rho A$$

$$I = \int \lambda r^2 dL$$

Area mass density:
Mass per unit area of a sheet of thickness L

$$\sigma = \frac{m}{A} = \rho L$$

$$I = \int \sigma r^2 dA$$

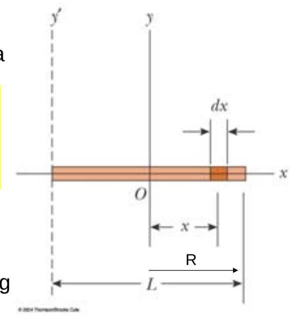
Moment of Inertia of a Uniform Rigid Rod

- The shaded area has a mass
  $dm = \lambda\, dx$
- Then the moment of inertia is

$$I = \int r^2 dm = \int_{-L/2}^{L/2} x^2 \frac{M}{L} dx$$

$$I = \frac{1}{12} ML^2 = \frac{1}{3} MR^2$$

Note: Careful about the choice of origin. That should be the point of rotation (except when using the parallel axis theorem)



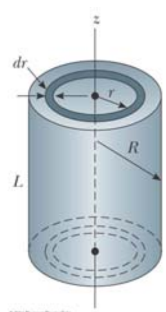
Moment of Inertia of a Uniform Solid Cylinder

- Divide the cylinder into concentric shells with radius *r*, thickness *dr* and length *L*
- Then for *I*

$$I = \int r^2 dm = \int r^2 \left(2\pi\rho L r\, dr\right)$$

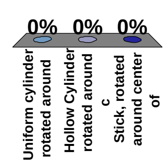
$$I_z = \frac{1}{2} MR^2$$

DEMO



Consider three objects **of the same max distance from rotation axis and same mass**. Which has the smallest moment of inertia?

1. Uniform cylinder rotated around cylinder axis
2. Hollow Cylinder rotated around cylinder axis
3. Stick, rotated around center of stick (not along stick axis)



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