Finding Stable Periodic Orbits in Families of One-Dimensional Maps

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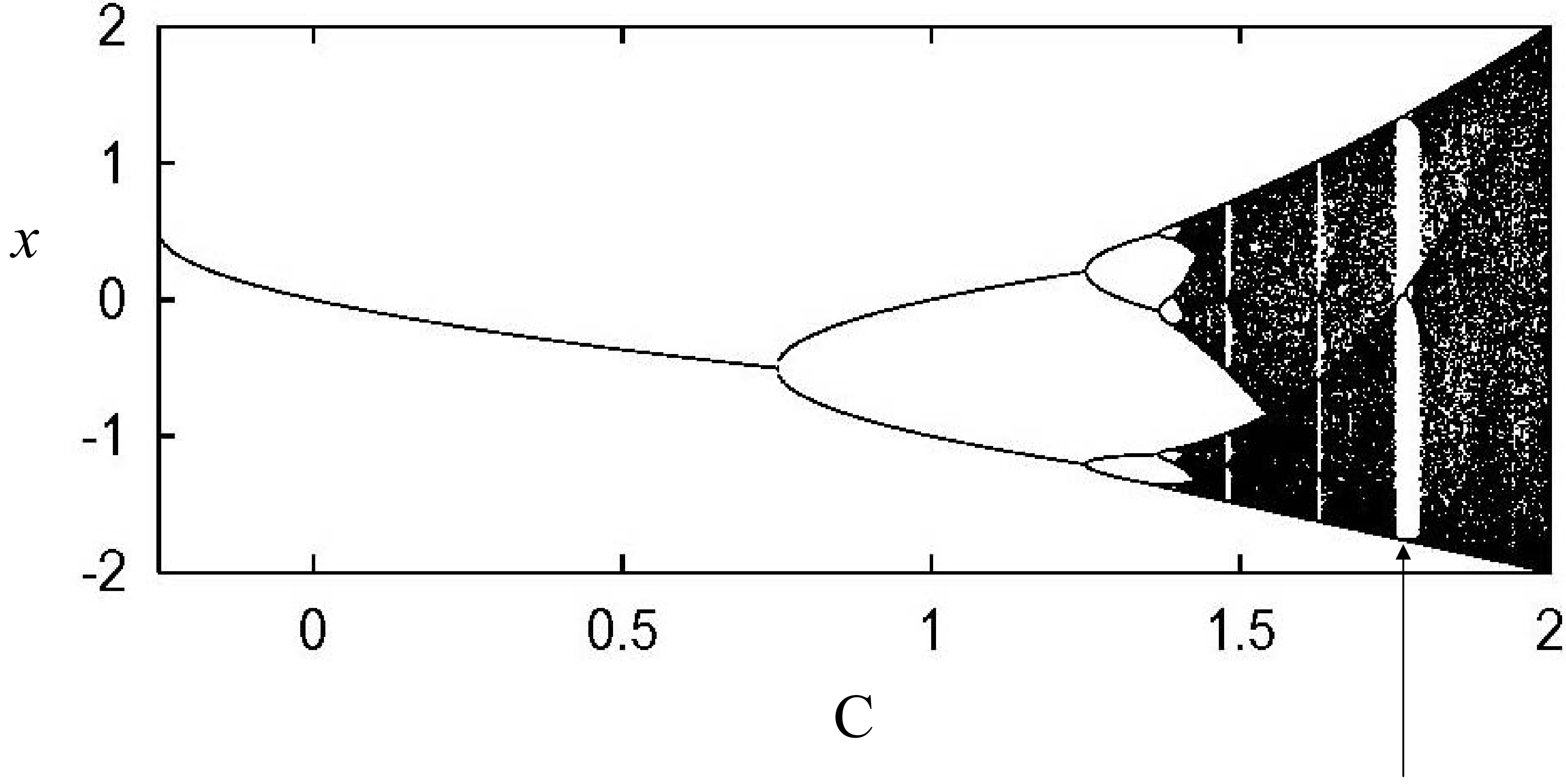
Stable Periodic Windows in Bifurcation Diagrams

Bifurcation Diagram for the Quadratic Family: $x_{n+1} = x_n^2 - C$



Period 3 Window: 1.75 < C < ~1.77

The Goal

- Given the ability to generate trajectories of a map, $x_{n+1} = f_C(x_n)$, for various values of C, find a nearby value of C for which the map has a stable point x of period p
- A period p point is stable if the derivative of the p^{th} iterate has absolute value less than 1

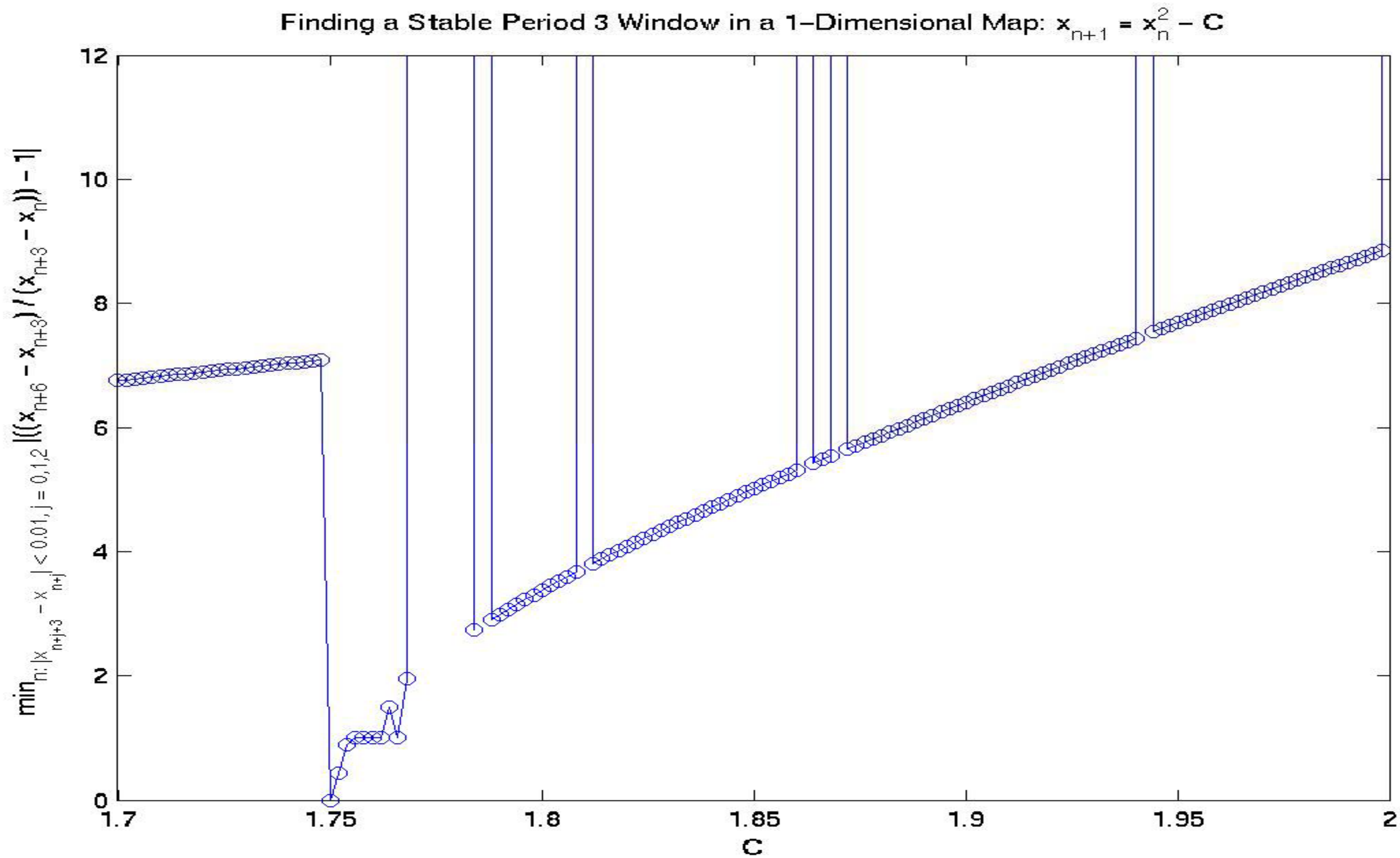
Using the Derivative

- We compute an approximate derivative of the pth iterate of the map on the periodic orbit when the orbit is approximately periodic:

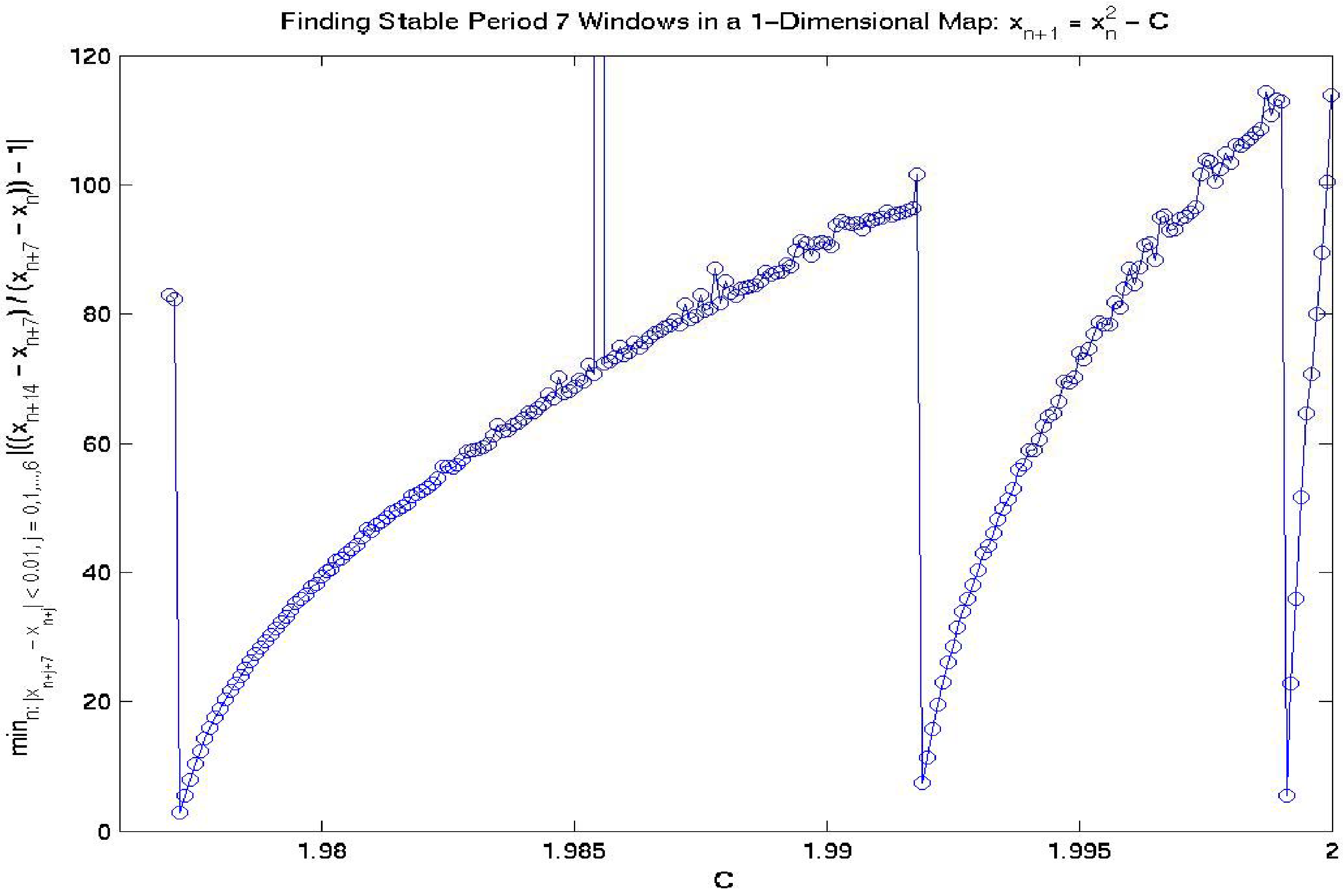
$$\left| \frac{f_C^p(x_{n+p}) - f_C^p(x_n)}{x_{n+p} - x_n} \right| = \left| \frac{x_{n+2p} - x_{n+p}}{x_{n+p} - x_n} \right|$$

- We do not need knowledge of the underlying maps to calculate this approximation – only data from generating a trajectory.

Approximate Derivative of the Third Iterate



Finding Period 7 Windows



Conclusion: An Algorithm For Finding Windows

- Computing an approximate derivative over a fine mesh of points effectively identifies proximity to windows (but is computationally intensive)
- Since we expect that the derivative is approximately of the order $\alpha\sqrt{C - C_0}$, we can fit a curve of this form to a *small* sample of data $\rightarrow$ we only have to compute an approximate derivative at a few points
- Where this fitted curve hits 0 provides an estimate for the location of the window
- The curve also provides an estimate of the size of the window
- Thus we have a feasible method to find windows