# WAVE CHAOS

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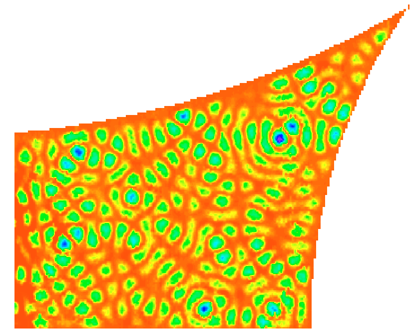
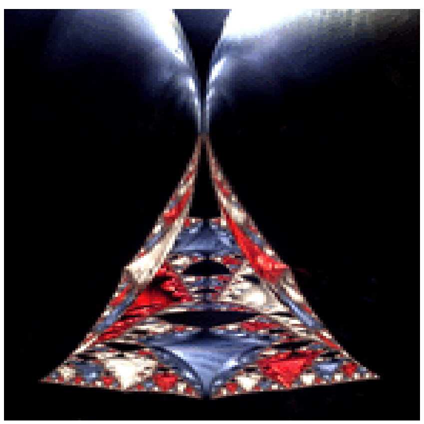
# CHAOS and NONLINEAR DYNAMICS at the UNIVERSITY of MARYLAND

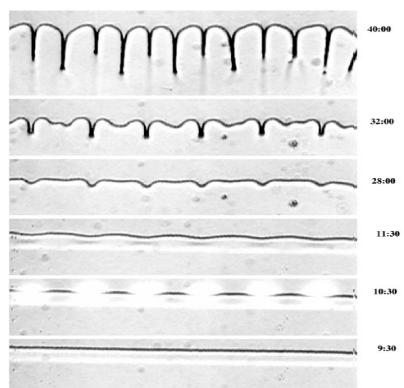
- **Balanced program involving collaboration among engineers, physicists, and mathematicians**
  - **-Basic Theory**
  - **-Applications**
  - **-Experiments**



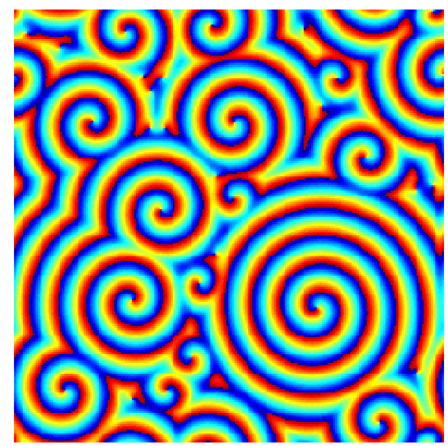
**Quantum chaos**



**Chaotic Scattering**



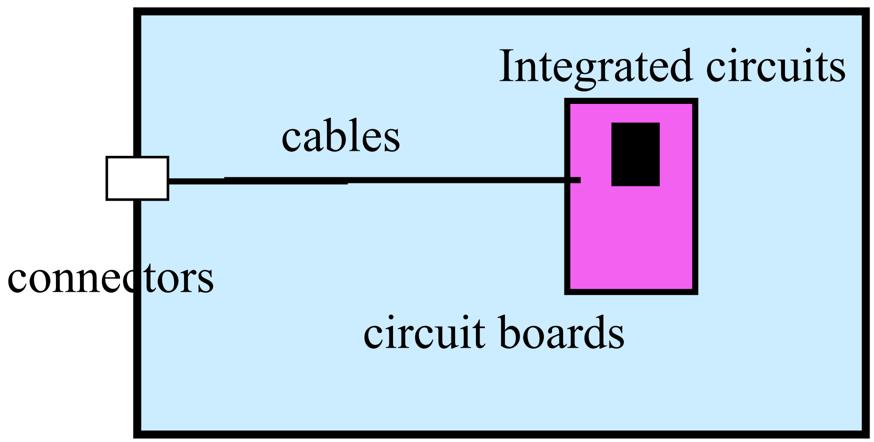

**Guided microstructure formation in binary alloys**



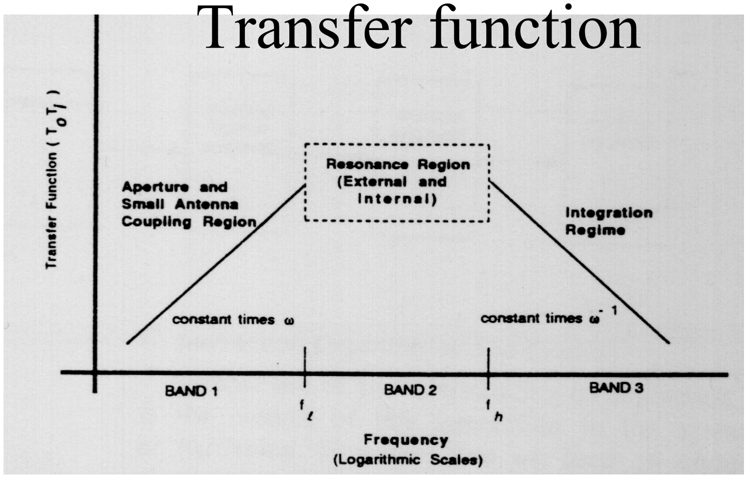
**Spiral patterns**

# Electromagnetic Coupling in Computer Circuits

## Schematic




## Transfer function




C. E. Baum, Proc. IEEE 80, 789-817 (1992).

- Coupling of external radiation to computer circuits is a complex processes:
  - apertures
  - resonant cavities
  - transmission lines
  - circuit elements
- Intermediate frequency range involves many interacting resonances
- What can be said about coupling without solving in detail the complicated EM problem ?
- Wave Chaos

# WAVE CHAOS
# (Also called "Quantum Chaos")

<u>Basic Question:</u> What happens to the solution of a wave equation (Maxwell's Eqs. or Schrödinger Eq.) when the solutions of the ray equations are chaotic?

## <u>Ray Equations:</u>

$$\omega = \omega(\boldsymbol{k}, \boldsymbol{x})$$

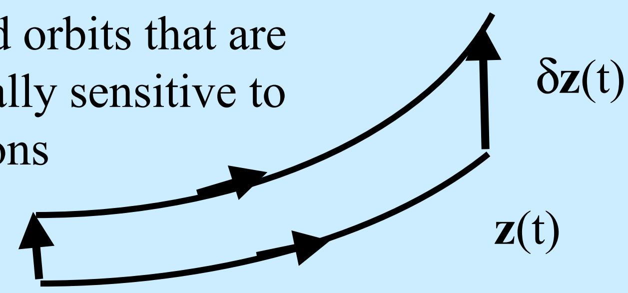
$$\frac{d\boldsymbol{x}}{dt} = \frac{\partial \omega(\boldsymbol{k}, \boldsymbol{x})}{\partial \boldsymbol{k}}$$

$$\frac{d\boldsymbol{k}}{dt} = -\frac{\partial \omega(\boldsymbol{k}, \boldsymbol{x})}{\partial \boldsymbol{x}}$$

## <u>Chaos:</u>

$$\frac{dz(t)}{dt} = \boldsymbol{F}(z(t))$$

Typical initial conditions lead to bounded orbits that are exponentially sensitive to perturbations

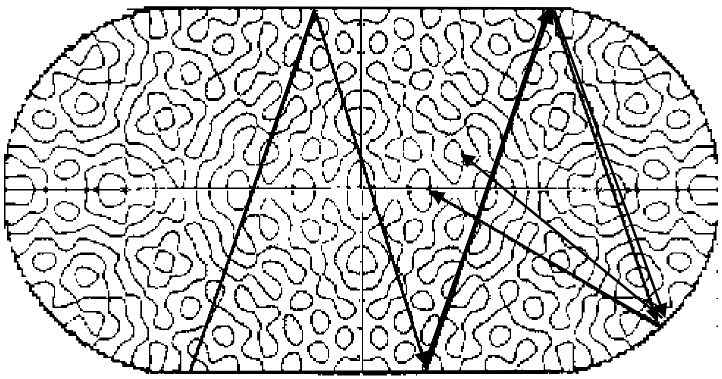


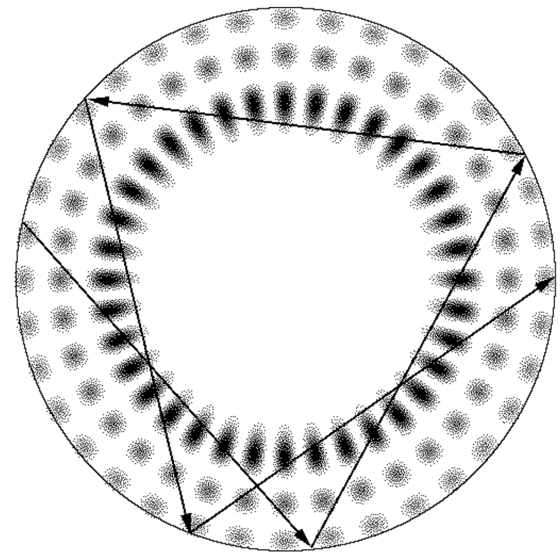

$$\delta z(t) \sim \delta z(0) \exp(ht)$$

h (Lyapunov exponent) > 0

# Examples

$$\nabla^2 \psi_n + k_n^2 \psi_n = 0$$



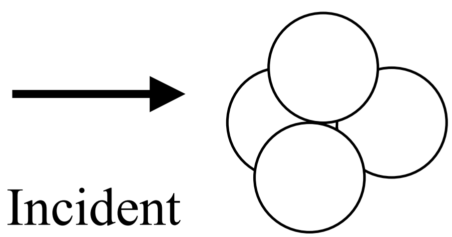



Stadium:
Trajectories are chaotic

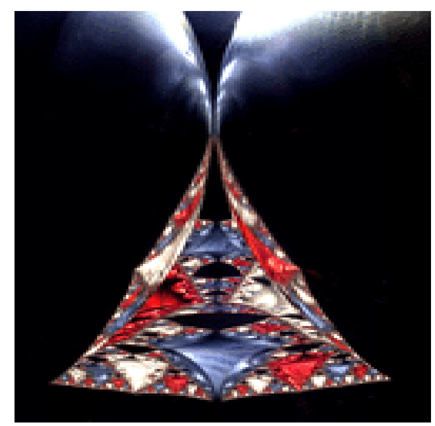
Circle:
trajectories are not chaotic

# Issues Addressed by Wave Chaos Theory

- Eigenvalue distributions
- Statistical properties of eigenfunctions
- Ray splitting
- Role of time reversal symmetry
- Chaotic scattering



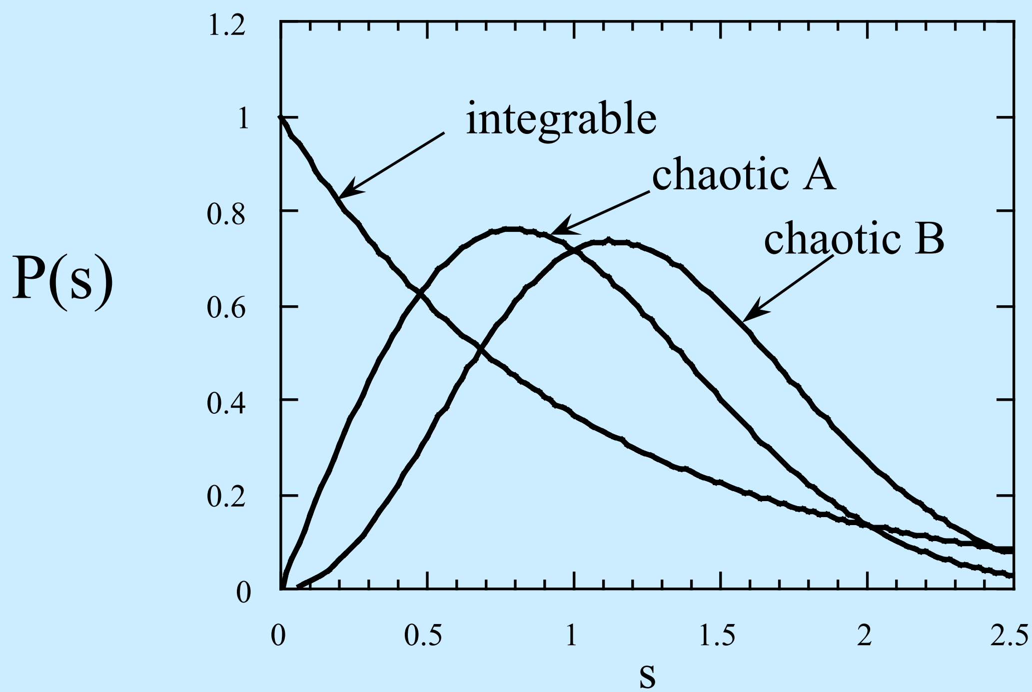
Incident wave



**Chaotic Scattering**

# Distribution of Eigenvalue Spacings

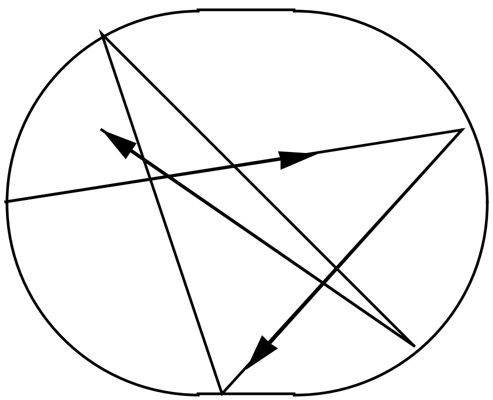
Normalized Spacing $s_n = (k_{n+1}^2 - k_n^2)/\left\langle \Delta k^2 \right\rangle$




A: time reversible

B: time irreversible
(e.g., due to ferrite)

# Properties of Eigenfunctions



Rays ergodically fill phase space.

Eigenfunctions appear to be a superposition of plane wave with random amplitudes and phases.

$$\psi_\alpha = \lim_{N\to\infty} \mathrm{Re}\left\{\sum_{j=1}^{N} a_j \exp\left[i\left(k_j \cdot x + \alpha_j\right)\right]\right\}$$ reversible

$$\psi_\alpha = \lim_{N\to\infty} \sum_{j=1}^{N} a_j \exp\left[i\left(k_j \cdot x + \alpha_j\right)\right]$$ irreversible

$|\mathbf{k}_j| = k_\alpha$, directions of $\mathbf{k}_j$ randomly distributed

$a_j$ random amplitude, $\langle a_j^2 \rangle = 1$

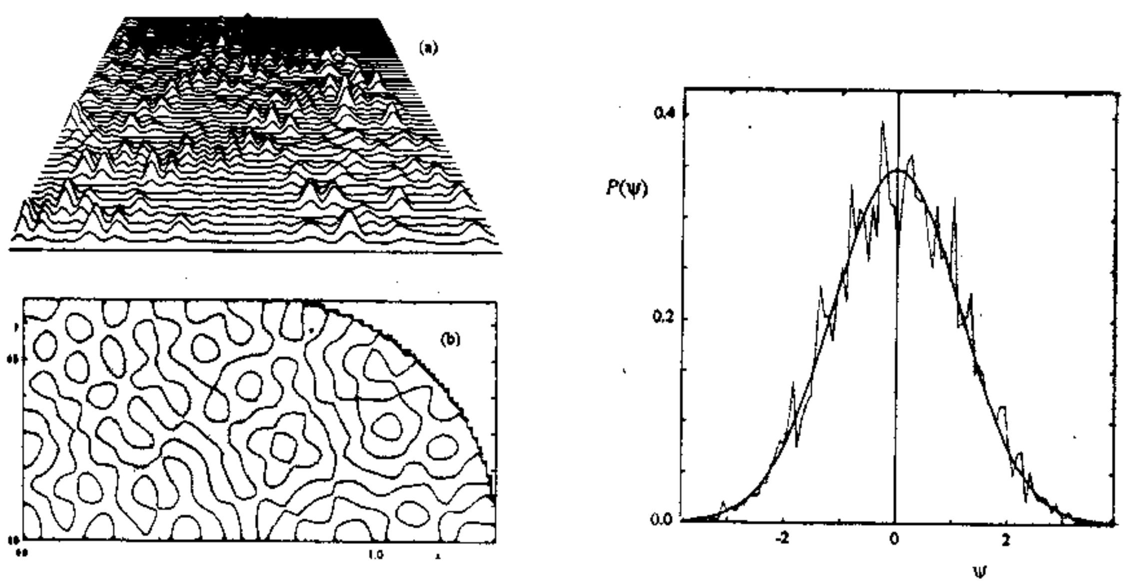
$\alpha_j$ random phase

$\psi_\alpha$ is a Gaussian random variable

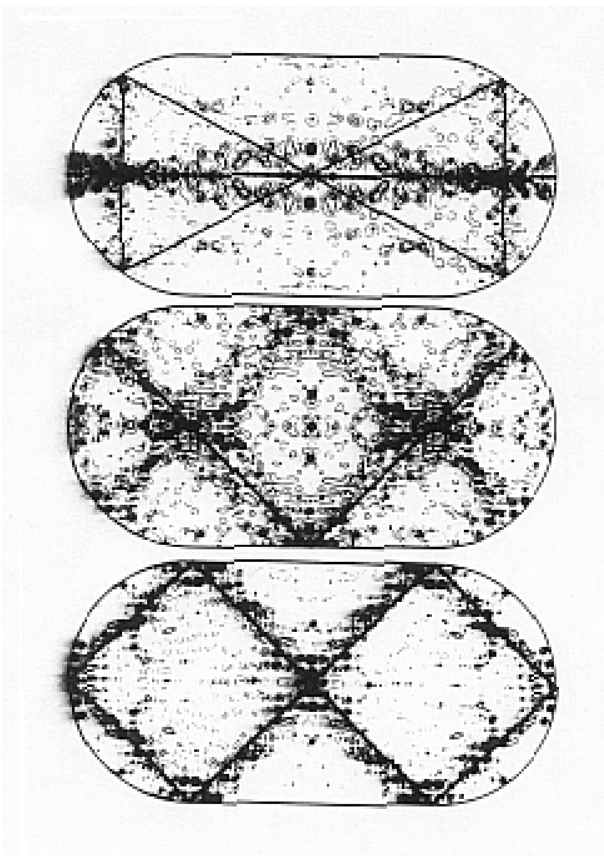
$$P(\psi) \sim \exp\left[-|\psi|^2 / 2\left\langle|\psi|^2\right\rangle\right]$$

# Numerical Verification of Random Eigenfunctions

Numerical Verification (McDonald and Kaufman 1988)



# SCARS – Exceptional Eigenfunctions



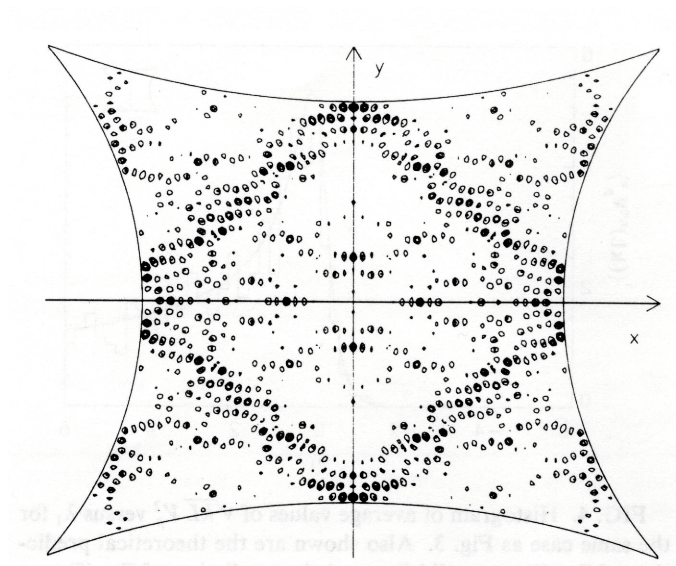
SCARS (Heller, 1984)

Concentrations of wave density along unstable periodic orbits.

Wave counterpart to classical phase space density is not uniform.

# STATISTICS of SCARS

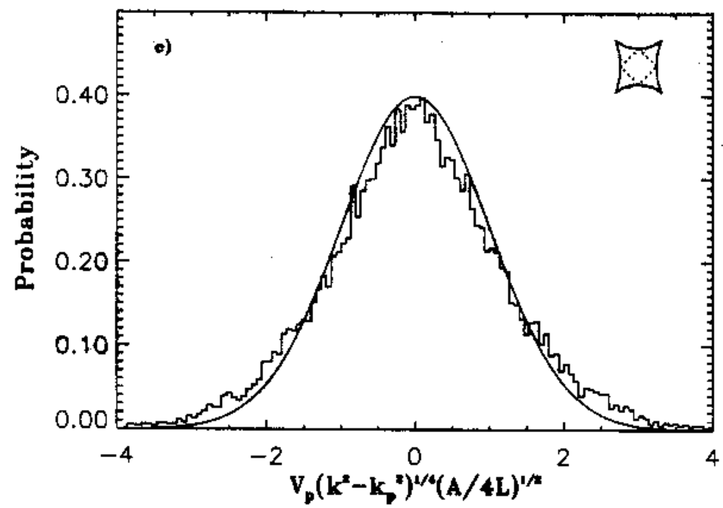
- At a specified frequency, what is the PDF of the scar strength?
- What periodic orbits will have the biggest scars?




Bow-Tie with diamond scar

# Statistics of Scars

T. Antonsen, E. Ott, Q. Chen and R.Oerter, Phys. Rev. E (1995).

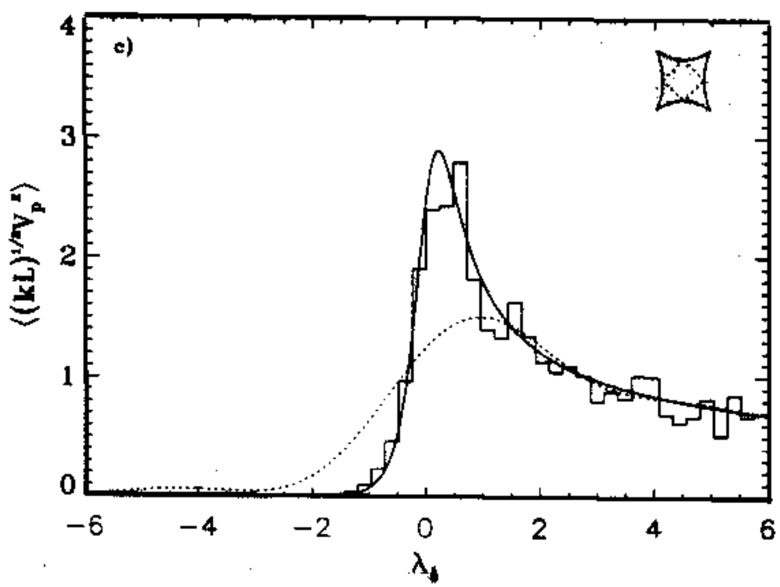



PDF of Scar Strength
Still Gaussian

# Statistics of Scars

Average Scar Strength

Strength of Scar on Periodic orbit

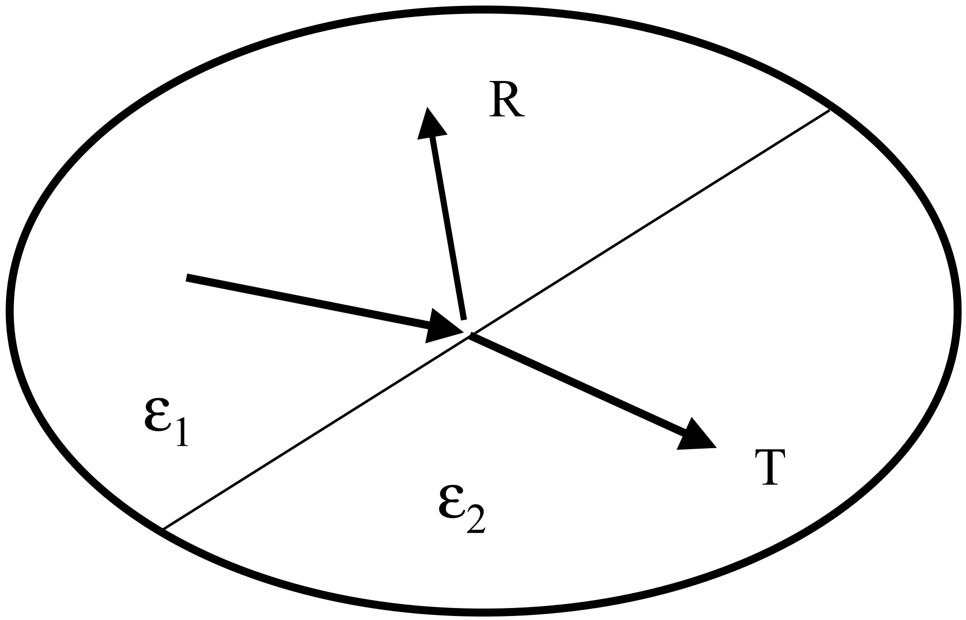



$$\lambda_1 = 2(k_n - \frac{p\pi}{L})L/(\Lambda - 1)$$

$k_n$ – eigenvalue
L – length of orbit
p – integer
$\Lambda$ - Lyapunov exponent

# RAY SPLITTING

Branching of rays at discontinuities



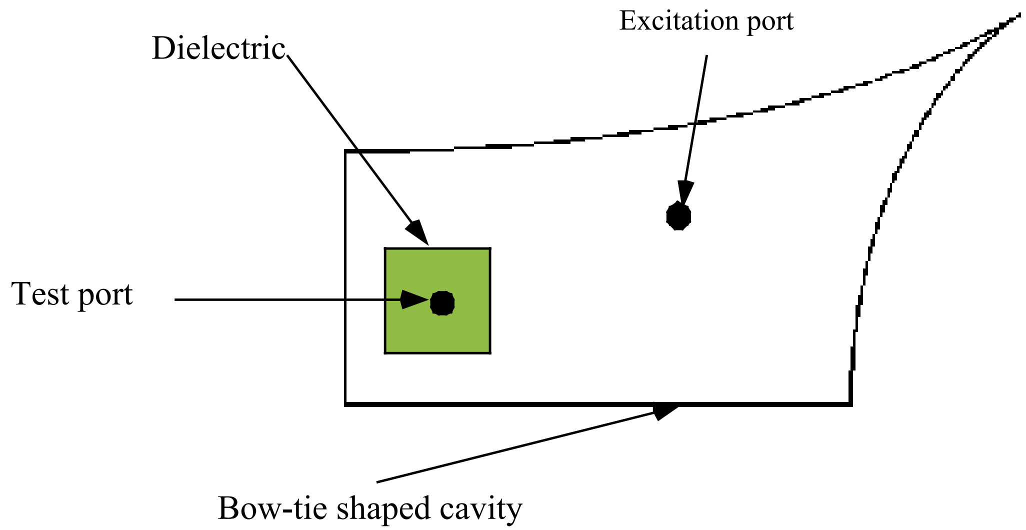

Increased chaos – fewer scars?

R. Blumel, T. Antonsen, B. Georgeot, E. Ott, and R. Prange, Phys. Rev. E (1996).

# Questions to be Addressed by this Project

- What are the properties of scars in 3D systems?
- What is the effect of ray splitting on scars?
- Can the wave chaos picture be extended to systems consisting of both propagating waves and circuit elements?
- What is the role of losses?
- What is the role of time reversal symmetry on wave functions?

# Possible First Problem

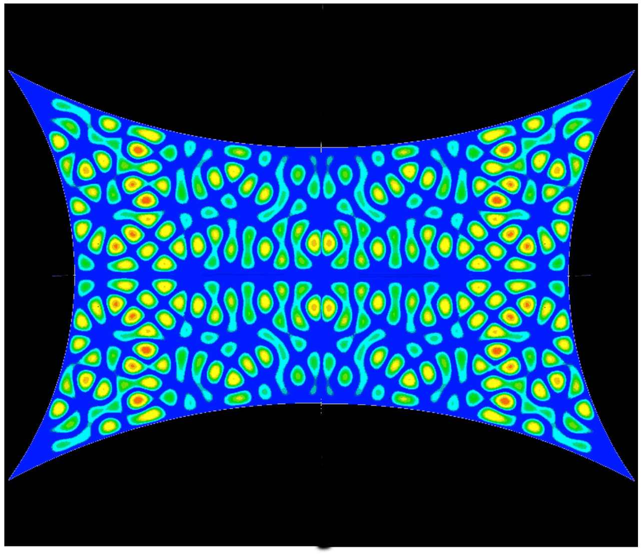



Features:

Ray splitting
Losses

Additional complications to be added later

- Can be addressed
  - -theoretically
  - -numerically
  - -experimentally



HFSS simulation courtesy J. Rodgers