Linear Theory of a Gyrotwystron with Stagger-Tuned Cavities

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***Abstract*— Gyrotwystrons with a large number of stagger-tuned prebunching cavities are promising wide-band high-gain amplifiers of electromagnetic waves. In the present paper, a small-signal theory describing the tradeoff in the gain and the bandwidth in these devices is developed. The results of the study of gyrotwystrons with one-, two-, and three-cavity prebunching sections are presented. These results show a significant increase in the bandwidth due to the stagger-tuning.**

***Index Terms*—Bandwidth, gain, stagger-tuning, gyrotwystron.**

I. Introduction

THE gyrotwystron, like the conventional twystron, consists of an input cavity and an output waveguide separated by a drift region. In a more complicated version of this device, instead of one cavity and one drift region there can be a set of prebunching cavities separated by drift regions. Like in klystrons and gyroklystrons, the microwave field of prebunching cavities causes weak modulation in electron energies. In drift regions, this modulation leads to the orbital phase bunching of electrons in the same way as in gyroklystrons. In the output waveguide, this prebunched electron beam excites electromagnetic (EM) waves, i.e., the interaction between electrons and the EM field in this section is similar to that in gyro-traveling-wave-tubes (gyro-TWT's).

Such a hybrid-type device is capable of combining the high efficiency and gain of klystrons and gyroklystrons and the large bandwidth of TWT's and gyro-TWT's, which makes gyrotwystrons promising amplifiers of coherent EM radiation. Since smooth-wall waveguides used in fast-wave gyrodevices can operate in a wide frequency range, the bandwidth of gyrotwystrons is mainly restricted by the Q-factor of the input cavity (or Q-factors of prebunching cavities). To overcome this restriction one can use a set of prebunching cavities with slightly detuned eigenfrequencies. This method, known as stagger tuning, is widely used in conventional klystrons for both bandwidth enlargement and efficiency enhancement (see [1] and [2] and references therein) as well as in gyroklystrons [3]–[7]. In the same way, in gyrotwystrons the stagger tuning of prebunching cavities can make the bandwidth of the prebunching section of a tube close to the bandwidth of the output waveguide section, thus making the bandwidth of the device much larger than in the absence of stagger tuning.

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The theory of gyrotwystrons has been developed in [8]–[14]. Also, the designs of C-band and W-band gyrotwystrons were presented, respectively, in [15] and [16], and the experiments with C-band and X-band gyrotwystrons were described in [17] and [18], respectively. In some of these papers it was shown that the bandwidth of gyrotwystrons can significantly exceed that of gyroklystrons; note the 1.5% bandwidth experimentally realized in [17], which is much larger than the 0.1–0.4% bandwidth typical for gyroklystrons.

In spite of a large number of the above-mentioned papers containing important information about gyrotwystrons, a more or less general analysis of the gain and bandwidth properties of gyrotwystrons with stagger-tuned multicavity prebunching sections, to the best of our knowledge, has not yet been done.

In the present paper we develop a linear theory of such devices in the way which allows us to analytically study the tradeoff in the gain and bandwidth of stagger-tuned gyrotwystrons. Our formalism is based on the approach used earlier in the development of the theory of stagger-tuned gyroklystrons ([19]). The paper is organized as follows: in Section II we present a general formalism; in Section III we present the results of the study; Section IV contains a discussion of the results; and Section V is the summary.

II. General Formalism

As mentioned above, the process of electron prebunching in a set of stagger-tuned cavities in gyrotwystrons is the same as in gyroklystrons. For gyroklystrons this process is described in detail elsewhere [19]. Therefore, in the present paper we will focus on the operation of the output waveguide, although the prebunching section will also be considered below.

A. Output Waveguide

The interaction between gyrating electrons and electromagnetic waves in a waveguide, as was shown in earlier studies [e.g., [11], [20], [21]], can be described by a self-consistent set of equations for the slowly variable normalized energy w and phase θ of electrons and the wave amplitude F

$$\frac{dw}{d\zeta} = -2\frac{(1-w)^{s/2}}{1-bw}\,\mathrm{Re}\Big(Fe^{-i\theta}\Big), \tag{1}$$

$$\frac{d\theta}{d\zeta} = \frac{1}{1-bw}\Big[w-\Delta' + s(1-w)^{(s/2)-1}\,\mathrm{Im}\Big(Fe^{-i\theta}\Big)\Big] \tag{2}$$

$$\frac{dF}{d\zeta} = -I\frac{1}{2\pi}\int_0^{2\pi}\frac{(1-w)^{s/2}}{1-bw}e^{i\theta}\,d\theta_0. \tag{3}$$

Here, $w = 2[(1-h\beta_{z0})/\beta_{\perp 0}^2][(\gamma_0-\gamma)/\gamma_0]$ (where $\beta_{\perp 0}$ and β_{z0} are, respectively, the orbital and axial components of the initial electron velocity normalized to the speed of light, γ is the electron energy normalized to the rest energy, and h is the axial wavenumber k_z normalized to ω/c); and $\theta = s\Theta - (\omega t - k_z z)$ is the phase of the sth harmonic of electron oscillations with respect to the phase of the traveling wave ($\Theta = \int_0^\tau \Omega\, d\tau' + \Theta_0$ is the gyrophase, Ω is the electron cyclotron frequency, and τ is the time of electron interaction with the wave; $\tau = 0$ corresponds to the entrance to the output waveguide). The normalized axial coordinate ζ in (1)–(3) is equal to

$$\zeta = \frac{\beta_{\perp 0}^2}{2\beta_{z0}} \frac{1-h^2}{1-h\beta_{z0}} \frac{\omega z}{c}. \tag{4}$$

$b = h\beta_{\perp 0}^2/2\beta_{z0}(1-h\beta_{z0})$ is the parameter responsible for the changes in the electron axial velocity in the process of radiation/absorption of EM waves, and $\Delta' = 2(1-h\beta_{z0})(1-h\beta_{z0} - s\Omega_0/\omega)/\beta_{\perp 0}^2(1-h^2)$ is the normalized cyclotron resonance mismatch. The normalized beam current parameter I in (3) is proportional to the beam current I_b

$$I = \frac{eI_b}{m_0 c^3}\left(\frac{2\beta_{z0}}{\beta_{\perp 0}^2}\frac{1-h\beta_{z0}}{1-h^2}\right)^2 \frac{1-h\beta_{z0}}{\gamma_0\beta_{z0}^2} \times \left[\frac{1}{(s-1)!2^s}\left(\frac{\kappa s\beta_{\perp 0}}{1-h\beta_{z0}}\right)^{s-1}\right]^2 2\frac{\kappa^2}{h}\frac{J_{m\mp s}^2(k_\perp R_0)}{(\nu^2-m^2)J_m^2(\nu)}. \tag{5}$$

Equation (5) is written for the cylindrical geometry of the interaction space and a thin annular electron beam interacting with a $\mathrm{TE}_{m,p}$ wave (corresponding expression for arbitrary geometries can be found in [20] and [21]), κ is the transverse wavenumber $k_\perp$ normalized to ω/c, R_0 is the electron guiding center radius, and ν is the pth root of the equation $J_m'(\nu) = 0$ which is the boundary condition for the $\mathrm{TE}_{m,p}$ wave at the waveguide wall. The fact that we use the first-order equation for F (3) instead of the second-order equation which follows directly from the wave equation implies that the device operates at frequencies far enough from cutoff, thus the effect of nonsynchronous opposite wave can be neglected. At the same time, below, in order to simplify our treatment, we will assume that

$$h^2,\ h\beta_{z0} \ll 1$$

and correspondingly $b \to 0$. Note that there is no contradiction between these two assumptions because one can neglect the effect of the opposite wave on electrons; for instance, in the standing wave $\sin(q\pi\, z/L)$ (where L is the waveguide length) when the number of axial variations q is large enough: $q \gg 1$. At the same time, the requirement $h^2 \ll 1$ for this wave is equivalent to $(q\lambda/2L)^2 \ll 1$. So, at large L/λ ratios both of these conditions can be fulfilled.

In the frame of the small-signal theory the EM field causes only small perturbations in the electron motion which in the absence of this field is described by $w_{(0)} = 0$, $\theta_{(0)} = \theta_0 - \Delta'\zeta$. Taking this into account and introducing instead of perturbations in the electron energy $\tilde{w}$ and phase $\tilde{\theta}$ the corresponding variables averaged over initial gyrophases,

$$w' = \frac{1}{2\pi}\int_0^{2\pi} \tilde{w} e^{i\theta_{(0)}} d\theta_0, \quad \theta' = \frac{1}{2\pi}\int_0^{2\pi} \tilde{\theta} e^{i\theta_{(0)}} d\theta_0 \tag{6}$$

one can reduce (1)–(3) to the following set of linear differential equations:

$$\frac{dw'}{d\zeta} + i\Delta' w' = -F, \tag{7}$$

$$\frac{d\theta'}{d\zeta} + i\Delta'\theta' = w' - i\frac{s}{2}F, \tag{8}$$

$$\frac{dF}{d\zeta} = -I\left(i\theta' - \frac{s}{2}w'\right). \tag{9}$$

At the entrance to the output waveguide $F(0) = 0$ while $w'(0)$ and $\theta'(0)$ are nonzero due to prebunching (these values, $w'(0)$ and $\theta'(0)$, will be described below).

Assuming that all variables (w', θ', and F) depend on ζ as $\exp(i\lambda'\zeta)$ one can derive from (7)–(9) the following dispersion equations for $\tilde{\lambda} = \lambda' + \Delta'$:

$$\tilde{\lambda}^2(\tilde{\lambda} - \Delta') - sI\tilde{\lambda} + I = 0 \tag{10}$$

which has the same form as (106) in [20] and (30) in [21] for $b = 0$. When the beam current parameter is small we can introduce $\bar{\lambda} = \tilde{\lambda}/I^{1/3}$, $\Delta = \Delta'/I^{1/3}$ (correspondingly, the axial coordinate should be renormalized as $\bar{\zeta} = I^{1/3}\zeta$) and reduce (10) to the dispersion equation

$$\bar{\lambda}^2(\bar{\lambda} - \Delta) + 1 = 0, \tag{11}$$

well known in the theory of traveling-wave-tubes (see, e.g., [22]). This equation has three roots which dependence on Δ was studied by Pierce [23]. (Note that in our case $I^{1/3}$ plays the role of the Pierce parameter C.)

Representing the perturbation in electron energy as

$$w' = \sum_{i=1}^{3} C_i e^{i\lambda_i'\zeta}$$

one can derive from (7) and (8) corresponding expressions for θ' and F:

$$\theta' = i\frac{s}{2}\sum_{i=1}^{3}\left(1 - \frac{2}{s\tilde{\lambda}_i}\right) C_i e^{i\lambda_i'\zeta}, \tag{12}$$

$$F = -i\sum_{i=1}^{3} \tilde{\lambda}_i C_i e^{i\lambda_i'\zeta}. \tag{13}$$

At the entrance to the waveguide these perturbations obey the boundary conditions that give us

$$C_1 + C_2 + C_3 = w'(0) \tag{14a}$$

$$i\frac{s}{2}\left[\left(1-\frac{2}{s\tilde{\lambda}_1}\right)C_1 + \left(1-\frac{2}{s\tilde{\lambda}_2}\right)C_2 + \left(1-\frac{2}{s\tilde{\lambda}_3}\right)C_3\right] = \theta'(0), \tag{14b}$$

$$\tilde{\lambda}_1 C_1 + \tilde{\lambda}_2 C_2 + \tilde{\lambda}_3 C_3 = 0. \tag{14c}$$

This set of linear algebraic equations allows one to determine the amplitudes of partial waves C_i as the functions of roots of the dispersion equation (10) (which depend on Δ' and I) and the functions of boundary values $w'(0)$ and $\theta'(0)$, which depend on the prebunching.

When the waveguide length is long enough we can take into account at the waveguide exit only the growing wave and neglect the decaying and constant amplitude waves. This allows us to determine the wave intensity at the waveguide exit $(\zeta = \mu_w)$ as

$$|F_{\rm out}|^2 \simeq 4\left|\frac{\tilde{\lambda}D_3}{D}\right|^2 e^{2\lambda''\mu_w}. \tag{15}$$

Here $D = 2[\tilde{\lambda}_1\tilde{\lambda}_3(\tilde{\lambda}_1 - \tilde{\lambda}_3) + \tilde{\lambda}_1\tilde{\lambda}_2(\tilde{\lambda}_2 - \tilde{\lambda}_1) + \tilde{\lambda}_2\tilde{\lambda}_3(\tilde{\lambda}_3 - \tilde{\lambda}_2)]/s\tilde{\lambda}_1\tilde{\lambda}_2\tilde{\lambda}_3$ is the determinant of (14), $D_3 = (\tilde{\lambda}_2 - \tilde{\lambda}_1)\{w'(0)[1 - 2(\tilde{\lambda}_1 + \tilde{\lambda}_2)/\tilde{\lambda}_1\tilde{\lambda}_2] + i(2/s)\theta'(0)\}$ is the subdeterminant for the third partial wave which we assume growing with the increment λ''. This increment follows from the solution of (10) (or (11) if μ_w is also normalized to $I^{1/3}$).

Having presented this general formalism for the output waveguide let us consider the prebunching in stagger-tuned cavities.

B. Prebunching Section

As was mentioned above, the formalism describing the electron prebunching in a set of stagger-tuned cavities separated by drift regions was presented in [19] (see also [24]). The notations adopted in [19] are slightly different from what we use here in accordance with [21]. However, when the Doppler term $h\beta_{z0}$ is negligibly small (as we assumed above) and the electron axial momentum is constant, the electron orbital momentum normalized to its initial value p, which was used in [19], and the normalized energy w introduced above are related as $p = \sqrt{1-w}$. Also, the slowly variable phase ϑ used in [19] and the phase θ we use here are related as $\vartheta = -\theta/s$ and the normalized axial coordinate in [19] is s times smaller than ζ used above.

To simplify our consideration we will analyze only the operation of all prebunching cavities at the fundamental cyclotron harmonic (operation at arbitrary harmonics was considered elsewhere [24]). Then the perturbations in the normalized energy w and the phase θ ($\theta = \theta_{(0)} + \tilde{\theta}$ where $\theta_{(0)} = \theta_0 - \Delta'\zeta$) in the mth cavity obey the following equations:

$$\frac{d\tilde{w}}{d\zeta} = -2\,{\rm Re}(F_m f_m e^{-i\theta_{(0)}}), \tag{16}$$

$$\frac{d\tilde{\theta}}{d\zeta} = \tilde{w} + {\rm Im}(F_m f_m e^{-i\theta_{(0)}}). \tag{17}$$

Here $F_m = |F_m|e^{i\psi_m}$ is the complex amplitude of the mth cavity field, and the function $f_m(\zeta)$ describes its axial structure. Linear equations (16) and (17) can be easily integrated, which gives for the variables w' and θ' used in (12)–(15) the following boundary conditions:

$$w'(0) = -\sum_{m=1}^{M} |F_m||u_m|e^{i(\psi_m+\alpha_m)}, \tag{18}$$

$$\theta'(0) = -\sum_{m=1}^{M} |F_m||u_m|e^{i(\psi_m+\alpha_m)}\left(\mu_{m,M} + \frac{i}{2}\right). \tag{19}$$

Here $u_m \equiv |u_m|e^{i\alpha_m} = \int_0^{\mu_m} f_m e^{i\Delta\zeta}d\zeta$ describes the effect of the axial structure of the cavity field, and $\mu_{m,M}$ is the distance between the exit from the mth cavity and the entrance to the output waveguide. At this distance the electron bunching caused by energy modulation in the mth cavity proceeds. Substituting (18) and (19) in the subdeterminant D_3 given above one gets

$$D_3 = 2\frac{\tilde{\lambda}_2 - \tilde{\lambda}_1}{\tilde{\lambda}_1\tilde{\lambda}_2}\sum_{m=1}^{M}|F_m||u_m|e^{i(\psi_m+\alpha_m)} \times(\tilde{\lambda}_1 + \tilde{\lambda}_2 - i\tilde{\lambda}_1\tilde{\lambda}_2\mu_{m,M}). \tag{20}$$

The amplitudes and phases of the fields in all cavities obey the balance equations given elsewhere [19]. These equations for the first cavity where the input signal is introduced have the form

$$|F_1|^2 = \frac{A_s^2}{\left(1 - I_{01}\chi''_{1(0)}\right)^2 + \left(\delta_1 + I_{01}\chi'_{1(0)}\right)^2} \tag{21}$$

$$\tan\psi_1 = \frac{\delta_1 + I_{01}\chi'_{1(0)}}{1 - I_{01}\chi''_{1(0)}} \tag{22}$$

for all other cavities

$$|F_m|^2 = \frac{4I_{0m}^2|u_m|^2|\mathcal{F}_m|^2}{\left(1 - I_{0m}\chi''_{m(0)}\right)^2 + \left(\delta_m + I_{0m}\chi'_{m(0)}\right)^2} \tag{23}$$

$$\tan\psi_m = \frac{\delta_m + I_{0m}\chi'_{m(0)}}{1 - I_{0m}\chi''_{m(0)}}. \tag{24}$$

In (21) A_s^2 is the normalized intensity of the field excited by the input signal in the first cavity, I_{0m} in all these equations is the normalized current parameter,

$$\chi = \chi_{(0)} + \chi_{(1)}\mathcal{F} \tag{25}$$

is the susceptibility of the electron beam with respect to the cavity field. The first term $\chi_{(0)}$ in (25) describes the susceptibility in the absence of prebunching effects; the second term is responsible for prebunching: $\chi_1 = -2i|u|/|F|$, $\mathcal{F} = (1/2\pi)\int_0^{2\pi} e^{-i\theta(0)}d\theta_0$. In (21)–(24) $\delta_m = 2Q_m(\omega - \omega_m)/\omega$ is the frequency mismatch between the operating frequency ω (which is the signal frequency) and the cold-cavity frequency ω_m, Q_m is the cavity Q-factor. Below we will consider a "point-gap" model of the prebunching section, i.e., we will assume that we have very short cavities separated by long drift regions. For short cavities (see, e.g., [19]) $\chi_{(0)} \simeq -i$ and $u = 1$. Correspondingly, (21)–(24) can be rewritten as

$$|F_1|^2 = \frac{A_s'^2}{1 + \delta_1'^2} \tag{26}$$

$$\tan\psi_1 = \delta_1' \tag{27}$$

$$|F_m|^2 = \frac{4I_{0m}'^2|\mathcal{F}_m|^2}{1 + \delta_m'^2} \tag{28}$$

$$\tan\psi_m = \delta_m' \tag{29}$$

where $\delta_m' = \delta_m/(1 + I_{0m})$, $A_s' = A_s/(1 + I_{01})$, $I_{0m}' = I_{0m}/(1 + I_{0m})$. The term $|\mathcal{F}_m|^2$ in (28) for the point-gap model, in the frame of the small-signal theory, can be defined as

$$|\mathcal{F}_m|^2 = \left|\frac{1}{2\pi}\int_0^{2\pi} e^{-i(\theta_0 - \Delta\sum_{m'=1}^{m-1}\mu_{\mathrm{dr},m'})}(1 - i\tilde{\theta}_m(0))\,d\theta_0\right|^2 = |\theta'_m(0)|^2$$

where $\mu_{\mathrm{dr},m'}$ is the length of the m'th drift section. In accordance with (19), when we assumed that in a point-gap model the field amplitude in each successive cavity is much larger than in the preceding one, we get

$$|\mathcal{F}_m|^2 \simeq |F_{m-1}|^2\mu_{\mathrm{dr},m-1}^2. \tag{30}$$

The equations derived allow us to analyze the small-signal gain in the multicavity stagger-tuned gyrotwystron.

C. Gain

We will analyze the gain determined as the ratio of the intensity of the output radiation to the intensity of the field exciting the input cavity

$$G = 10\log\left(\frac{|F_{\mathrm{out}}|^2}{|A_s|^2}\right) \tag{31}$$

assuming, as above, that in a chain of prebunching cavities for any m, $|F_m|^2 \gg |F_{m-1}|^2$. This allows us after representing the ratio $|F_{\mathrm{out}}|^2/A_s^2$ as the product of ratios

$$\frac{|F_{\mathrm{out}}|^2}{|F_M|^2}\frac{|F_M|^2}{|F_{M-1}|^2}\cdots\frac{|F_1|^2}{A_s^2} \tag{32}$$

to get for any ratio $|F_m|^2/|F_{m-1}|^2$ from (28) and (30) the following equation:

$$\frac{|F_m|^2}{|F_{m-1}|^2} = \frac{4I_{0m}'^2\mu_{\mathrm{dr},m-1}^2}{1+\delta_m'^2}. \tag{33}$$

Correspondingly, (20) is reduced to

$$D_3 = 2\frac{\tilde{\lambda}_2 - \tilde{\lambda}_1}{\tilde{\lambda}_1\tilde{\lambda}_2}|F_M|e^{i\psi_M}(\tilde{\lambda}_1 + \tilde{\lambda}_2 - i\tilde{\lambda}_1\tilde{\lambda}_2\mu_{\mathrm{dr},M}). \tag{34}$$

where $\mu_{\mathrm{dr},M}$ is the distance between the last cavity and the output waveguide, i.e., the length of the last drift section. When this drift section is long enough and $\tilde{\lambda}_1 \sim \tilde{\lambda}_2 \sim 1$

$$D_3 \simeq -2i(\tilde{\lambda}_2 - \tilde{\lambda}_1)|F_M|\mu_{\mathrm{dr},M}e^{i\psi_M}. \tag{35}$$

Therefore, the first ratio in the chain given by (32) can be represented, as follows from (15) and (35), as

$$\frac{|F_{\mathrm{out}}|^2}{|F_M|^2} = 16\mu_{\mathrm{dr},M}^2 e^{2\lambda''\mu_w}\left|\frac{\tilde{\lambda}_3(\tilde{\lambda}_2 - \tilde{\lambda}_1)}{D}\right|^2. \tag{36}$$

Substituting (32), (33), and (36) into (31) we get

$$G = 10\log\left\{\Phi(\Delta)I^{4/3}\left(\prod_{m=1}^{M}\frac{\mu_{\mathrm{dr},M}^2}{1+\delta_m'^2}\right)\prod_{m=2}^{M}I_{0m}'^2\right\} \tag{37}$$

where in accordance with (36)

$$\Phi(\Delta) = 4\left|\frac{\bar{\lambda}_3(\bar{\lambda}_2 - \bar{\lambda}_1)}{D}\right|^2 e^{2\gamma\bar{\mu}_w}. \tag{38}$$

In (38) $\gamma\bar{\mu}_w = \lambda''\mu_w$, i.e., γ is the increment of the growing wave which can be found from (11). We also made transition from $\tilde{\lambda}$ [in (36)] to $\bar{\lambda}$, taking into account that the determinant D remains unchanged; thus the factor $I^{4/3}$ appeared in (37). The maximum value of Φ corresponds to the exact cyclotron resonance when $\Phi(\Delta = 0) = \Phi_{\max} = (4/9)e^{\sqrt{3}\bar{\mu}_w}$.

So, in the absence of stagger tuning and in the case of exact cyclotron resonance in the output waveguide the small-signal gain is maximum and equal to

$$G_{\max} = 10\log\left\{\Phi_{\max}\left(\prod_{m=1}^{M}\mu_{\mathrm{dr},m}^2\right)I^{4/3}\left(\prod_{m=2}^{M}I_{0m}'^2\right)\right\}. \tag{39}$$

Note that this gain is linearly proportional to the normalized length of the output waveguide while as the function of the lengths of the drift sections it changes in a logarithmic scale.

In the presence of stagger tuning and for nonzero cyclotron resonance mismatches the gain can be represented as $G_{\max} - G_{\mathrm{var}}$ where the variable term G_{var} as follows from (37) and (39), is equal to

$$G_{\mathrm{var}} = G_w + \sum_{m=1}^{M}G_m. \tag{40}$$

Here

$$G_w = 10\log\{\Phi_{\max}/\Phi(\Delta)\} = 10\log\left\{9\left|\frac{\bar{\lambda}_3(\bar{\lambda}_2 - \bar{\lambda}_1)}{D}\right|^2 e^{(\sqrt{3}-2\gamma)\bar{\mu}_w}\right\} \tag{41}$$

describes the effect of the cyclotron resonance mismatch Δ on the small-signal gain in the output waveguide and

$$G_m = 10\log[1 + \bar{Q}_m(\Delta - \Delta_m)^2] \tag{42}$$

describes the effect of Δ and stagger tuning on the gain in the mth cavity. To unify notations for prebunching cavities and the output waveguide we introduced in (42) $\bar{Q}_m = \beta_{\perp 0}^2 I^{1/3}Q_m/(1 + I_{0m})$ and $\Delta_m = (2/\beta_{\perp 0}^2 I^{1/3})[(\omega_m - \Omega_0)/\omega - h\beta_{z0}]$. Now the same detuning is present in G_w where Φ and γ depend on Δ and in G_m. The cavity gain G_m depends also on the stagger-tuning parameter Δ_m. In the limiting case of vanishingly small Q-factors of all cavities, $G_{\mathrm{var}} = G_w$. However, for realistic situations the opposite case, $\bar{Q}_m \gg 1$ takes place. In this case the bandwidth of the device is restricted by the cavities' bandwidth. To avoid this restriction, the stagger-tuning should be used [nonzero Δ_m's in (42)]. Correspondingly, the gain, as follows from (40), becomes smaller. This tradeoff in the gain and bandwidth will be illustrated below by consideration of several simple schemes.

III. Results

Let us start our analysis from consideration of the waveguide gain function G_w determined by (41). This function is shown in Fig. 1 as the function of the cyclotron resonance mismatch Δ for three values of the normalized length of the waveguide: $\bar{\mu}_w = 4, 6$, and 8. Certainly, when the waveguide

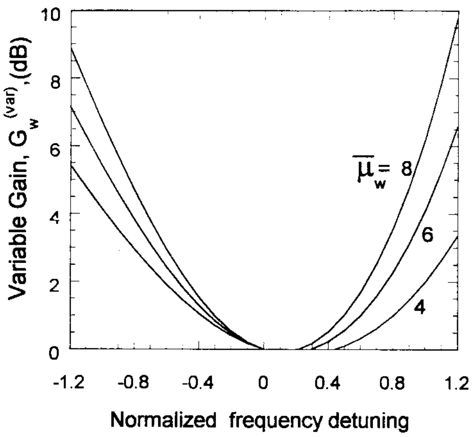


Fig. 1. The variable gain in the output waveguide for several values of the normalized waveguide length as a function of the normalized frequency detuning Δ.

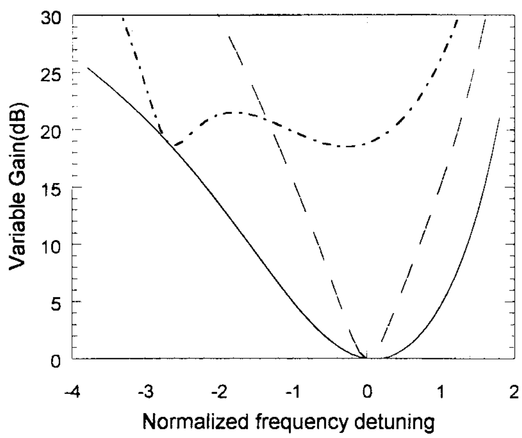


Fig. 2. The variable gain in a gyrotwystron with a one-cavity prebunching and $\bar{\mu}_w = 6$. The solid line shows the case of an infinitely low $\bar{Q}_1$. The dashed line shows the case of $\bar{Q}_1 = 3.2$ in the absence of stagger-tuning. The dash-dotted line shows the case of $\bar{Q}_1 = 3.2$ with the maximum stagger-tuning.

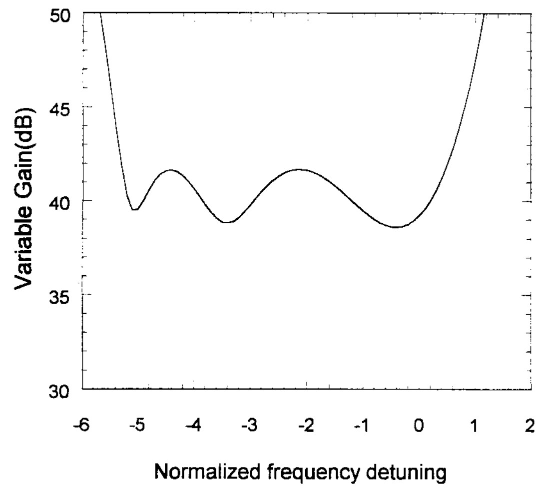


Fig. 3. The variable gain in a gyrotwystron with a two-cavity prebunching (parameters $\bar{Q}_1 = 1.5$, $\bar{Q}_2 = 3.5$, $\Delta_1 = -3.4$, and $\Delta_2 = -5.1$ correspond to the maximum bandwidth).

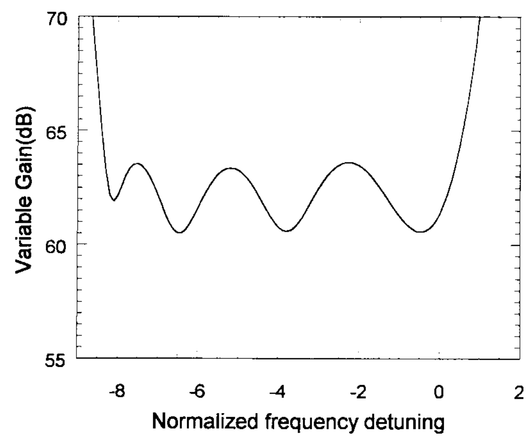


Fig. 4. The variable gain in a gyrotwystron with a three-cavity prebunching and parameters providing the maximum bandwidth.

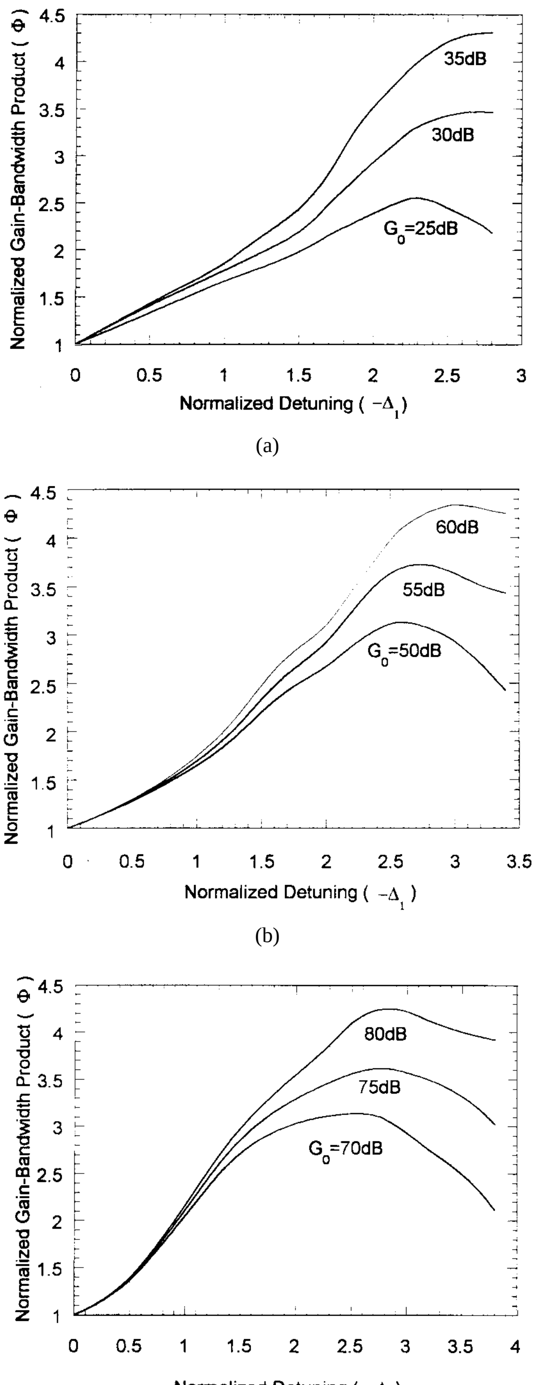
is shorter, the bandwidth is wider; however, the absolute value of the gain, as follows from (39), becomes smaller.

Fig. 2 illustrates the effect of the cavity Q-factor on the bandwidth of the gyrotwystron with a one-cavity prebunching. Here the solid line shows the variable gain in the case of negligibly small normalized Q-factor of the cavity ($\bar{Q}_1 \to 0$). (This is the same waveguide gain function G_w as shown in Fig. 1.) The dashed line shows the variable gain for a nonzero $\bar{Q}_1$ ($\bar{Q}_1 = 3.2$) but in the absence of stagger tuning, $\Delta_1 = 0$. This condition, $\Delta_1 = 0$, implies the cold cavity frequency ω_1 to be equal to the wave frequency corresponding to the exact cyclotron resonance $\omega_1 = \omega_{\rm res} = \Omega_0 + k_z v_{z0}$. As follows from Fig. 2, the bandwidth in this case is almost three times smaller than for $\bar{Q}_1 \to 0$. The dash-dotted line in Fig. 2 shows the variable gain for the gyrotwystron with the same value of $\bar{Q}_1$ but with the stagger tuning providing the maximum bandwidth ($\Delta_1 = -2.7$). The bandwidth in this case is more than two times larger than that determined by the waveguide properties (the solid line), however the gain is smaller by 20 dB.

Fig. 3 shows the variable gain in the gyrotwystron with a two-cavity prebunching. The cavities' parameters are chosen to provide the maximum bandwidth: $\bar{Q}_1 = 1.5$, $\bar{Q}_2 = 3.5$, $\Delta_1 = -3.4$, $\Delta_2 = -5.1$. As follows from a comparison of Fig. 3 with Fig. 2 (the normalized waveguide length here is the same), the bandwidth in the gyrotwystron with stagger-tuned two-cavity prebunching can be 1.65 times larger than that in the case of a stagger-tuned one-cavity prebunching, and 3.65 times larger than that determined by the output waveguide. However, the corresponding stagger tuning causes a reduction in gain by 40 dB.

A similar situation occurs in the gyrotwystron with a three-cavity prebunching. The bandwidth of this device is illustrated by Fig. 4 which corresponds to the set of cavity parameters providing the maximum bandwidth: $\bar{Q}_1 = 1.2$, $\bar{Q}_2 = 1.48$, $\bar{Q}_3 = 3.15$, $\Delta_1 = -3.8$, $\Delta_2 = -6.5$, and $\Delta_3 = -8.22$. In this case, the bandwidth is more than 1.5, 2.5, and 5.6 times larger, respectively, than in the cases of two-cavity prebunching, one-cavity prebunching, and a pure waveguide interaction. However, the maximum stagger tuning causes a reduction in gain by approximately 60 dB.

The gain-bandwidth product in stagger-tuned gyroamplifiers can reach its maximum when the stagger tuning is smaller



(a)

(b)

(c)

Fig. 5. Normalized gain-bandwidth product as a function of the stagger-tuning parameter for (a) a one-cavity gyrotwystron, (b) a two-cavity gyrotwystron, and (c) a three-cavity gyrotwystron.

than that which corresponds to the maximum bandwidth. For stagger-tuned gyroklystrons, it was shown in [19]. For stagger-tuned gyrotwystrons this statement is illustrated by Fig. 5. In Fig. 5(a) the ratio of the gain-bandwidth product in the stagger-tuned device to that in the absence of stagger tuning:

$$\Phi = \frac{\mathrm{BW}_{\mathrm{st}} G_{\mathrm{st}}}{\mathrm{BW}_0 G_0}$$

is shown for a one-cavity gyrotwystron with $\bar{Q}_1 = 3.2$ and several values of G_0 [G_0 is the maximum gain given by (39)] as the function of the detuning Δ_1. In Fig. 5(b) the same ratio is shown for the two-cavity gyrotwystron with $\bar{Q}_1 = 1.5$ and $\bar{Q}_2 = 3.2$ (these values of $\bar{Q}$ correspond to the maximum bandwidth; see Fig. 3) and a fixed ratio of Δ_1 to Δ_2, $\Delta_1/\Delta_2 = 2/3$ (again, when $\Delta_2 = -5.1$ this ratio gives the maximum bandwidth). Finally, Fig. 5(c) illustrates the dependence of the gain-bandwidth product on the stagger tuning in the three-cavity gyrotwystron. Again, $\bar{Q}$-values are the same as for the maximum bandwidth shown in Fig. 4 and the ratio of detunings Δ_1, Δ_2, and Δ_3 is also the same as in Fig. 4 (so for $\Delta_3 = -8.22$ we get the case shown in Fig. 4). As follows from Fig. 5, the gain-bandwidth product in stagger-tuned gyrotwystrons can be several times larger than in the absence of stagger tuning.

IV. Discussion

The bandwidth calculated above was given in terms of the normalized frequency detuning

$$\Delta = \frac{1}{I^{1/3}} \frac{2}{\beta_{\perp 0}^2} \frac{\omega - k_z v_{z0} - s\Omega_0}{\omega}.$$

This means that the bandwidth given in Δ, BW_Δ relates to the conventional bandwidth $\mathrm{BW} = \Delta f/f$ (where $f = \omega/2\pi$) as

$$\mathrm{BW}_\Delta = \frac{2}{\beta_{\perp 0}^2 I^{1/3}} \mathrm{BW}. \tag{43}$$

If we assume that a thin annular 60-kV, 5-A electron beam with the orbital-to-axial velocity ratio 1.5 excites the TE_{01} wave in the output waveguide at the fundamental cyclotron harmonic and the radius of electron guiding centers corresponds to the maximum coupling to the wave, then we get $\beta_{\perp 0}^2 = 0.138$ and $I^{1/3} \simeq 0.2/h^{1/3}$. Correspondingly, from (43) we get $\mathrm{BW} \simeq 0.0138\mathrm{BW}_\Delta/h^{1/3}$. So, for instance, when $h = 0.1$ it gives us $\mathrm{BW} \simeq 0.03\mathrm{BW}_\Delta$, thus the bandwidths of one-, two-, and three-cavity stagger-tuned gyrotwystrons shown in Figs. 2–4 correspond, respectively, to 10%, 17%, and 26%. In the absence of stagger tuning, the bandwidth of the gyrotwystron with a one-cavity prebunching, as follows from Fig. 2, is equal to only 1.6% (when $\bar{Q}_1 = 3.2$). Certainly, all these numbers serve only for illustrative purposes since we neglected above the electron velocity spread. Note that the effect of velocity spread on the locking bandwidth of phase-locked two-cavity gyrotrons was studied in [25], where it was shown that the 20% spread causes significant shrinkage of the bandwidth when the external magnetic field in the interaction region is nontapered; in the case of a properly tapered magnetic field in the drift region the effect is much weaker.

Before closing this section let us note that in our paper we did not study the excitation of parasitic backward waves in the output waveguide. The excitation of these waves in gyro-traveling-wave-tubes and gyrotwystrons was studied, respectively, in [26] and [27] (see also references therein). In particular, in [26] it was shown that the drive signal can increase the starting current of parasitic backward waves at least six times (in comparison with the starting current in the absence of the drive power). This issue as well as the large signal operation of gyrotwystrons with stagger-tuned multicavity prebunching will be analyzed later.

V. Conclusion

The small-signal theory of gyrotwystrons with an arbitrary number of stagger-tuned prebunching cavities was developed. The analysis of these devices with one, two, and three prebunching cavities operating at the fundamental cyclotron resonance was carried out. The results obtained demonstrate a significant bandwidth enlargement due to the stagger tuning and allow one to analyze the tradeoff in the gain and bandwidth in these tubes.

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Gregory S. Nusinovich (SM'92), for a biography, see this issue, p. 460.

Wenjun Chen, for a photograph and biography, see this issue, p. 460.

V. K. Tripathi, photograph and biography not available at the time of publication.